Math 6 Unit 1

LearnEveryWare

Student Learning Guide

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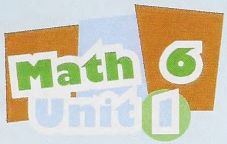

Math 6 Learn EveryWare – Unit 1 Student Learning Guide
ISBN: 978-0-7741-3097-4

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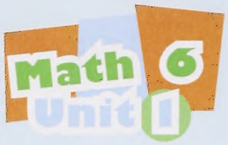

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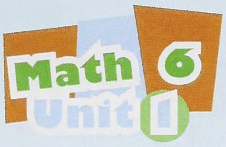
Etraffic Press® would like to acknowledge the production and project teams of Etraffic Solutions™ Inc. and The Distributed Learning Resources Branch for their contributions to the project management, design, editing and development of this publication.



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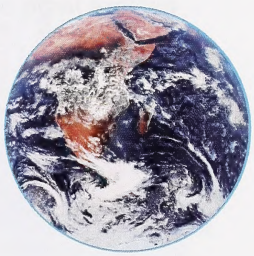
Lesson 1

Place Value

Population Explosion

The World Factbook estimates the 2008 world population to be about 6.7 billion people. This number can be written many ways:



In Words	six billion seven hundred million
In Standard Form	6 700 000 000
In Expanded Form	6 000 000 000 + 700 000 000

Using the birth and death rates from the World Factbook we can figure out how the population changes. About three people are added to the population every second. That is 180 people in one minute:

In Words	one hundred eighty
In Standard Form	180
In Expanded Form	100 + 80

Three people every second is 10 800 people every hour:

In Words	ten thousand eight hundred
In Standard Form	10 800
In Expanded Form	10 000 + 800

This means that every day there are 259 200 people added to the world population:

In Words	two hundred fifty-nine thousand two hundred
In Standard Form	259 200
In Expanded Form	200 000 + 50 000 + 9 000 + 200

These numbers used to describe population growth are very large. You will explore large and small numbers in this lesson.

Reflection

What are some other real life situations that involve large numbers?

Objectives for this Lesson

In this lesson you will explore the following concepts:

- Use the place value system to read large and small numbers
- Provide examples of how large and small numbers are used

- Go online to complete the Concept Capsule: Place Value to 100 000.

Place Value Charts

You need to be able to read and write large or small numbers. The base-ten number system creates an order to place values. The **periods** and order help you with reading the numbers to the left of the decimal.

You should recall the use of place value charts from earlier grades:

Millions				Thousands				Ones			.	Decimals					
Hundred Millions	Ten Millions	Millions		Hundred Thousands	Ten Thousands	One Thousands		Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths		Ten Thousandths	Hundred Thousandths

Each period to the left of the decimal is characterized by three place values. For example, the thousands period starts with one thousands then has ten thousands and hundred thousands. The millions period has the same pattern.

The following names are used, in this order, after the millions place:

billions, trillions, quadrillions, quintillions, and so on.

Reflection

What do you think the period for the trillions would look like?

- Go online to watch the Notepad Tutor Lesson: Place Value to 100 000 000.

If you look at the decimals, they follow a similar pattern. This pattern goes in the opposite direction. Within the decimals period you can almost see the forming of the pattern.

The pattern goes like this: tenths, hundredths, thousandths, ten thousandths, hundred thousandths, millionths, ten millionths, hundred millionths, and so on.

The possible place values go on and on and never end.

Understanding the pattern to the place value chart will help you identify numbers that are extremely large or very small.

Now It's Your Turn

Write these standard form numbers in word form.

a. 12 300 000 000 b. 320 200 100 c. 200 000.003

Solutions

a. twelve billion three hundred million

b. three hundred twenty million two hundred thousand one hundred

c. two hundred thousand and three thousandths

- Go online to complete the Concept Capsule: Super Large Numbers.

Writing Numbers

Writing numbers in expanded form is another way you can pick up on the pattern of the place value system.

Example 1

Write the number shown in the place value chart in standard form and expanded form.

Millions				Thousands				Ones			.	Decimals					
Hundred Millions	Ten Millions	Millions		Hundred Thousands	Ten Thousands	One Thousands		Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths		Ten Thousandths	Hundred Thousandths
3	2	0		5	1	6		4	7	0	.	0	8				

In **standard form** you should leave a space between periods, and place a decimal after the ones period.

320 516 470.08

For **expanded form** you should write the value of each digit other than 0 and add these values together:

300 000 000 + 20 000 000 + 500 000 + 10 000 + 6 000 + 400 + 70 + 0.08

Example 2

Write the number given in word form in standard form and expanded form.

three hundred fifty-eight million two hundred thirty thousand five hundred and fourteen ten thousandths

Take the words apart carefully as you read:

three hundred fifty-eight million

The "million" at the end tells you this is the end of the millions period.
You can think of this in short form: 358 million

two hundred thirty thousand

The "thousand" at the end of this phrase tells you this is the end of the thousands period. You can think of this as 230 thousand.

five hundred and fourteen ten thousandths

The number after "thousand" and before "and" (if there is a decimal) tells you your ones period. Think: 500
The "and" tells you this is the beginning of the decimal period.
The "ten thousandths" tells you the place value of the last digit.
The last digit of fourteen is (4) so it will go in the ten thousandths place.

Millions				Thousands				Ones			.	Decimals					
Hundred Millions	Ten Millions	Millions		Hundred Thousands	Ten Thousands	One Thousands		Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths		Ten Thousandths	Hundred Thousandths
											.			1		4	

The 1 has to go in the place before the 4 and the rest of the places after the decimal become 0:

.001 4

Now you need to put all that together:

Standard form:

358 230 500.001 4

Expanded form:

300 000 000 + 50 000 000 + 8 000 000 + 200 000 + 30 000 + 500 + 0.001 + 0.000 4

Example 3

The population of Canada is approximately thirty-three million, four hundred twenty-eight thousand nine hundred. Write the population of Canada in standard form and expanded form.

Break down the parts to write standard form first:

thirty-three million: 33 million

four hundred twenty-eight thousand: 428 thousand

nine hundred: 900

Standard form:

33 428 900

Expanded form:

30 000 000 + 3 000 000 + 400 000 + 20 000 + 8 000 + 900

You can also use place value to quickly subtract values.

Example 4

What is 10 000 less than 315 248 679?

Identify the number in the ten thousands place: 315 2**4**8 679

Ask yourself, "What is one less than this number?" 3

Now change that number to 3 and keep the rest of the number:

315 238 679

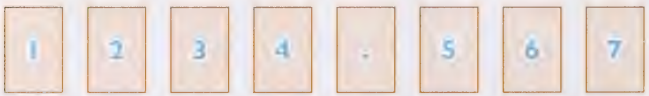
Place value can also be used to solve problems. Use the following Exploration to find ways to write small and large numbers.

Let's Explore

Exploration 1: Making Numbers

Materials: Unit 1, Lesson 1, Exploration 1 page in your Workbook, 8 Index Cards, Marker, Pencil

1. Write the following digits and the decimal on one index card each. Be sure to use your marker and make the numbers very large.



2. What is the largest number that you can make using these cards if you must have three digits after the decimal?
3. Write your large number in standard form.
4. Write your large number in expanded form.

5. Write your large number in words.
6. What is the smallest number that you can make using these cards if you must have only three digits after the decimal?
7. Write your small number in standard form.
8. Write your small number in expanded form.
9. Write your small number in words.
10. What is the largest number you can make if you can place the decimal anywhere you want? Write the number in standard form and in words.
11. What is the smallest number you can make if you can place the decimal anywhere you want? Write the number in standard form and in words.

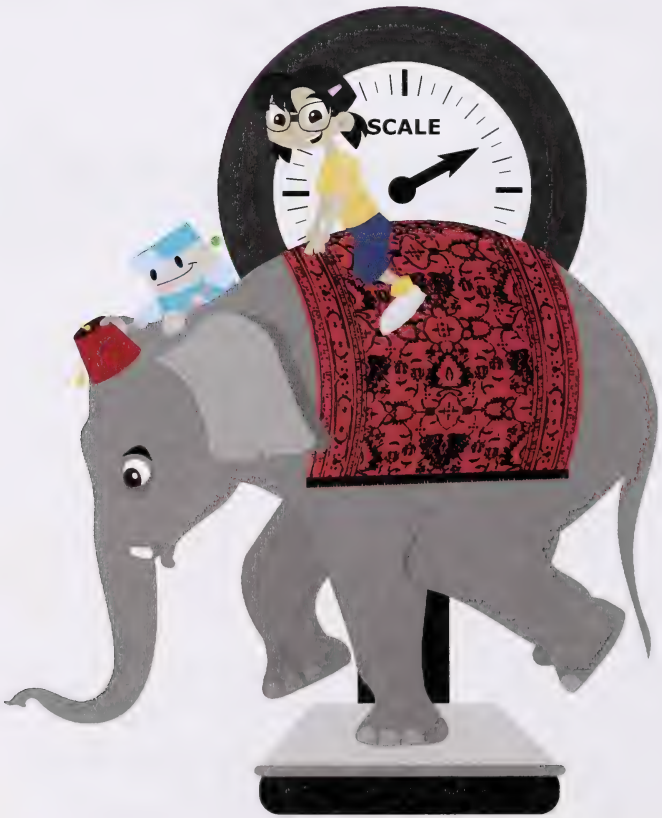
Let's Practice

- Turn in your Workbook to Unit 1, Lesson 1 and complete 1 to 18.

Numbers in Real Life

There are large numbers in many areas of science and in the media, such as:

Canada's retail trade for 2007	$412 000 000 000
Distance to the moon	384 403 km
Video game related sales for November 2006	$1 700 000 000
Average weight of an Asian elephant	5 500 kg



You can also find very small numbers, such as:

Average length of a carpenter ant	13 mm
Thickness of a penny	1.45 mm
Wavelength of green light	0.000 000 51 m

Let's Explore

Exploration 2: Searching for Numbers

Materials: Unit 1, Lesson 1, Exploration 2 page in your Workbook, Newspapers, Internet, Magazines, Science Textbooks, Pencil

1. Find five numbers in one of your resources that are greater than one million.
2. Record the source, what the number describes, and then write the number in standard form and expanded form in the table.
3. Find five decimal numbers that are smaller than one.
4. Record the source, what the number describes, and then write the number in standard form and expanded form in the table.
5. Reflect: What resources might you use to find large numbers?
6. Reflect: What resources might you use to find small numbers?

Let's Practice

- Turn in your Workbook to Unit 1, Lesson 1 and complete 19 to 22.

Lesson 2

Multiplying and Dividing Whole Numbers

Theme Park Attendance

There are many large theme parks in North America. These parks have thousands of visitors each day. The parks have rides and attractions that thrill you for hours. The following are some of the popular parks in North America:

Park	Yearly Attendance (2007)
Magic Kingdom at Walt Disney World, Lake Buena Vista, FL	17 060 000
Disneyland, Anaheim, CA	14 870 000
Canada's Wonderland, Maple, Ontario	3 250 000
Epcot at Walt Disney World, Lake Buena Vista, FL	10 930 000
Disney's Hollywood Studios at Walt Disney World, Lake Buena Vista, FL	9 510 000
Knott's Berry Farm, Buena Park, CA	3 630 000
Cedar Point, Sandusky, OH	3 120 000
Busch Gardens Tampa Bay, Tampa FL	4 400 000

Reflection

Which park do you think made the most money in one day? What kind of information would you need to figure that out?

3 600 450 people attended Canada's Wonderland in Maple, Ontario during one season. How many people attended the park each day on average if the park was only open for 127 days?

To solve a problem like this you must first decide the operation that will be used. You may already know that this has to be division, from the information given. The word "each" could have helped you figure that out.

$3\ 600\ 450 \div 127$

Seeing this problem, you may have noticed that you have never divided numbers this large before. You have probably used long division, but not with numbers this large. When numbers are this large you can use a calculator.

Using a calculator:

3 600 450 ÷ 127 = 28 350

This means that 28 350 people attended Canada's Wonderland on average each day.

Reflection

How many people do you think attended Disneyland in California each day? How could you find out?

Objectives for this Lesson

In this lesson you will explore the following concepts:

- Identify the operation used to solve a given problem
- Determine the reasonableness of an answer
- Determine if you should use technology
- Use technology when appropriate to solve a given problem

- Go online to watch the Notepad Tutor Lesson: Translating Word Problems.

Operations

There are four major **number operations** that you should be very familiar with by now:

addition, subtraction, multiplication, division

You should recall learning to use these operations with **whole numbers**. You also need to be able to know when to use each operation.

Here are some samples of story problems for each type of operation.

Addition	Subtraction
Nina had $12 and her aunt gave her $25 for her birthday. How much money does Nina have?	There are 3 245 seats in the baseball stadium. At the Friday night game 2 560 seats are filled. How many seats are empty?
Multiplication	**Division**
There are 250 rows of 20 seats in a stadium. How many seats are there in the stadium?	Zach has collected 1 314 marbles. He wants to put the same number of marbles in each of 12 jars. How many will be in each jar?

You should be able to determine the type of operation needed to solve various types of story problems. Use the following process to help you.

Example 1

Alyssa, Lian, and Cameron can create 438 cm of floral garland each day. They have two weeks to make garland for Alyssa's cousin's wedding. How many centimetres of garland can they make?

Read the problem and look for information. Make note of all clue words and relevant numbers in the problem.

438 cm
Each day
Two weeks (means 14 days)

Picture

Create a picture of the problem if you can. This can be a simple drawing or a table to help you determine the operation.

Day 1	438	**Day 8**	438
Day 2	438	**Day 9**	438
Day 3	438	**Day 10**	438
Day 4	438	**Day 11**	438
Day 5	438	**Day 12**	438
Day 6	438	**Day 13**	438
Day 7	438	**Day 14**	438

Plan

Think about the problem and what the question is asking.

There are 14 days and each day Alyssa, Lian, and Cameron make 438 cm of garland.

What is the total length of garland?

Possible methods to solve:

Repeated Addition	Multiplication
438 + 438 + 438 + 438 + 438 + 438 + 438 + 438 + 438 + 438 + 438 + 438 + 438 + 438 =	438 x 14 =

Solve

Repeated addition would take a really long time by hand. This is when you would want to use another method. Using multiplication is shorter and will reduce the possibility of error.

There are two methods you can use to multiply the numbers.

Multiply each digit of 438 by 4 by regrouping	Multiply each digit of 438 by 10	Add the two products together
$\overset{1\,3}{438}$ $\times\ 1\overset{}{4}$ $1\,752$	438 $\times\ 14$ $1\,752$ $+\ 4\,380$	438 $\times\ 14$ $1\,752$ $+\ 4\,380$ $6\,132$

You could also use the grouping by place value method:

Think of the expanded form of each number:

$438 = 400 + 30 + 8$
$14 = 10 + 4$

Multiply the value of each place in 438 by 4

Next, add the column.

$400 \times 4 = 1\,600$
$30 \times 4 = 120$
$8 \times 4 = 32$
$1\,752$

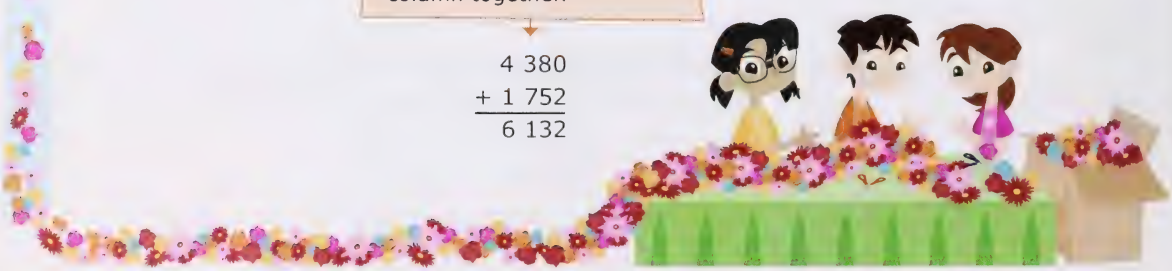
Multiply the value of each place in 438 by 10.

Next, add the column.

$400 \times 10 = 4\,000$
$30 \times 10 = 300$
$8 \times 10 = 80$
$4\,380$

Add the answers from each column together.

$4\,380$
$+\ 1\,752$
$6\,132$



Check

Ask yourself: "Is the answer reasonable?"

They made 6 132 centimetres of garland.

You can estimate the solution.

Using front-end estimation:

400 x 10 = 4 000

This is an estimate that is LOWER than the actual amount. Since the answer is more than this amount it is a reasonable estimate.

Making models can help you to decide which operation you use to solve a problem. Use models to solve the problems in the following exploration.

Let's Explore

Exploration 1: Making Models

Materials: Unit 1, Lesson 2, Exploration 1 page in your Workbook, Dry Beans (or Counters), Pencil

For 1- 4: Make a model using your materials. Sketch your model and write the problem and its solution.

1. Alyssa has 24 dry beans, Nina has 31 dry beans and Zach has 18 dry beans. How many dry beans do they have all together?
2. Cameron has 68 dry beans. He wants to give the same number to each of his 3 friends and himself. How many will each of them get?
3. Lian has 45 dry beans. She gives Daksha 26 dry beans. How many does Lian have left?
4. Nina has 18 dry beans. Cameron says he has five times as many dry beans as Nina. How many dry beans does Cameron have?

Is the Answer Reasonable?

You can use estimation strategies to estimate solutions before you solve using the full operation. This is how you can make sure your answer is reasonable.

Example 2

Daksha has earned $870 in the last six weeks by mowing lawns in his neighbourhood. How much did he make each week?

Estimate	Solution
$900 \div 6 = 150$ The answer should be about $150 per week.	$6\overline{)870}$ → 145 6 27 24 30 30 0

The answer "Daksha made $145 each week" is a reasonable answer.
The estimate is MORE than the actual answer since you rounded 870 up.
Since 145 is less than and close to the 150, you have a reasonable answer.

Let's Practice

- Turn in your Workbook to Unit 1, Lesson 2 and complete 1 to 8.

Technology with Operations

You should be able to do operations, like the following, by hand and without the use of a calculator:

Operation	Description	Example
Addition	Add up to four-digit numbers.	2 546 + 328
Subtraction	Subtract up to four-digit numbers.	5 487 – 849
Multiplication	Two-digit numbers by two-digit numbers.	43 x 56
Division	Three-digit numbers by one-digit numbers.	174 ÷ 3

If a problem involves going beyond the descriptions in this table, you may choose to use a calculator. Remember, the calculator does not solve a problem for you. It simply makes the calculation you tell it to make. If you tell it to divide when you should have used subtraction, your answer will be wrong.

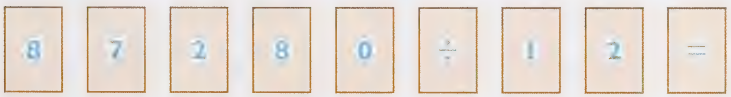
Example 3

Mr. Davis earned $87 280 salary for the year. How much did he make in one month?

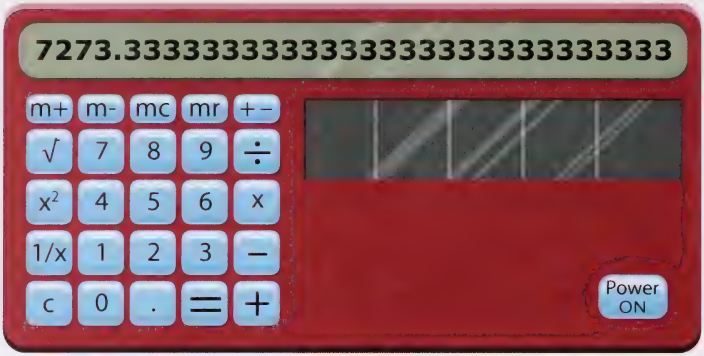
Write a problem:

87 280 ÷ 12

Use a calculator to divide. Type in the following keystrokes:



The calculator will return the following:



The solution will be in dollars and cents so you need to round to the hundredths place:

Mr. Davis earns $7 273.33 per month.

Using technology can help when the problem is complex or when you have really large numbers to calculate.

Let's Practice

- Go online to watch the Notepad Tutor Lesson: When and When Not to Use Technology.
- Turn in your Workbook to Unit 1, Lesson 2 and complete 9 to 15.

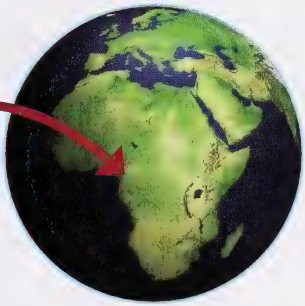
Lesson 3

Multiples of Whole Numbers

Helping People

Alyssa and Cameron are boxing rolled oats to send to hungry families in Africa. They place six canisters of rolled oats in each box for shipping.

Top View of Box



They pack boxes for five hours on a Saturday morning. What are the possible numbers of canisters that are ready to ship out when they are done?

Since each box holds six canisters the possible values are multiples of six.
The multiples of six are:
6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96, 102, 108, 114, 120...

Knowing the multiples of six can help you determine how many canisters of rolled oats have gone out.

After working for one hour packing boxes, Alyssa and Cameron have packed 20 boxes of rolled oats. How many canisters of rolled oats have they prepared?

The 20 boxes contain 6 canisters each. 20 x 6 = 120 canisters of rolled oats

Since the answer, 120, is on our list of multiples of 6 this is a reasonable answer for the number of canisters packed.

Reflection

How many canisters of rolled oats will Alyssa and Cameron have prepared by the end of 5 hours if they pack the same number each hour? The answer to this question would be a multiple of what number?

Objectives for this Lesson

In this lesson you will explore the following concepts:

- How to identify multiples for a given number
- Explain the strategy for finding the multiples of a given number
- Solve a given problem involving multiples of a number

Multiples

A **multiple** is a **product** of a given number and another number. If you want to know the multiples of ten, you would multiply 10 by every number, beginning with the number 1:

$$\begin{array}{rrrrrr} 10 & 10 & 10 & 10 & 10 & 10 \\ \times\ 1 & \times\ 2 & \times\ 3 & \times\ 4 & \times\ 5 & \times\ 6 \\ \hline 10 & 20 & 30 & 40 & 50 & 60 \end{array}$$

The first six multiples of 10 are 10, 20, 30, 40, 50, 60.

Example 1

What are the first eight multiples of 2?

You may know the multiples of 2 from having memorized them in earlier grades.

First eight multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16

If you do not know your multiplication tables, this is a good time to review your multiplication facts from 1 x 1 to 9 x 9.

Patterns in a Multiplication Table

It is easy to get to know the multiples of the numbers one to ten when you use a multiplication table. You can find the multiples of a number using the rows and columns of the table.

The first twelve multiples of 8 are highlighted in the multiplication table:

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

This helps you identify the first twelve multiples of any number from 1 to 12.

What are the first twelve multiples of 7?

Let's Explore

Exploration 1: Multiples of 7

Materials: Unit 1, Lesson 3, Exploration 1 page in your Workbook, Pencil

1. Colour the 7 row or the 7 column in the multiplication table.
2. List the first twelve multiples of 7 using this table.
3. Extend your list to include the next three multiples of 7.
4. Describe the method you used to find the next three multiples of 7.

Multiplication tables are also helpful when you are trying to solve problems with multiples.

Example 2

Decide if the statement is True or False and explain your reasoning.

Any multiple of 8 is also a multiple of 2.

Use the multiplication table to help you:

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

List the multiples of 2:

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24...

List the multiples of 8:

8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96...

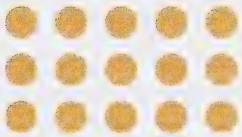
If you were to continue the list of multiples of 2, all of the multiples of 8 would be included.

The statement is TRUE.

Using Arrays to Find Multiples

You can use multiplication arrays to decide if a number is a multiple of a larger number. An **array** is a picture model to show the relationship between two multiples and their product.

3 rows of 5: 3 x 5 = 15



This array shows that 15 is a multiple of 3 and a multiple of 5. You can add a row of 5 to make a new array:

4 rows of 5: 4 x 5 = 20

This array shows that 20 is a multiple of 4 and a multiple of 5.

Let's Explore

Exploration 2: Multiples in Arrays

Materials: Unit 1, Lesson 3, Exploration 2 page in your Workbook, Counters (Small Candies or Dry Beans), Pencil

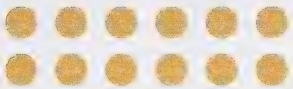
1. Make an array for 3 x 4 using Counters.
2. Use your Counters to create the array for each multiple of 4 up to the twelfth multiple of 4.
3. Did you add rows or columns to your original array?
4. How many Counters did you add each time?
5. Complete this statement: To get the 20^{th} multiple of 4 I would use ___ rows of ___ Counters.
6. Make an array for 2 x 6 using Counters.
7. Describe how you would change the array to create an array for a product of 36. Draw a picture of your new array.
8. Reflect: How many arrays can be made for the multiples of 8, out of 56 Counters?

A **square number** is a number that is formed by an array that makes a square. For example:

4 rows of 4: 4 x 4 = 16

The square number represented by this array is 16.

9. What is a square number that would have a row of 7 Counters?
10. The following array does not represent a square number. Add Counters to the array to create a square number. What number did you create?

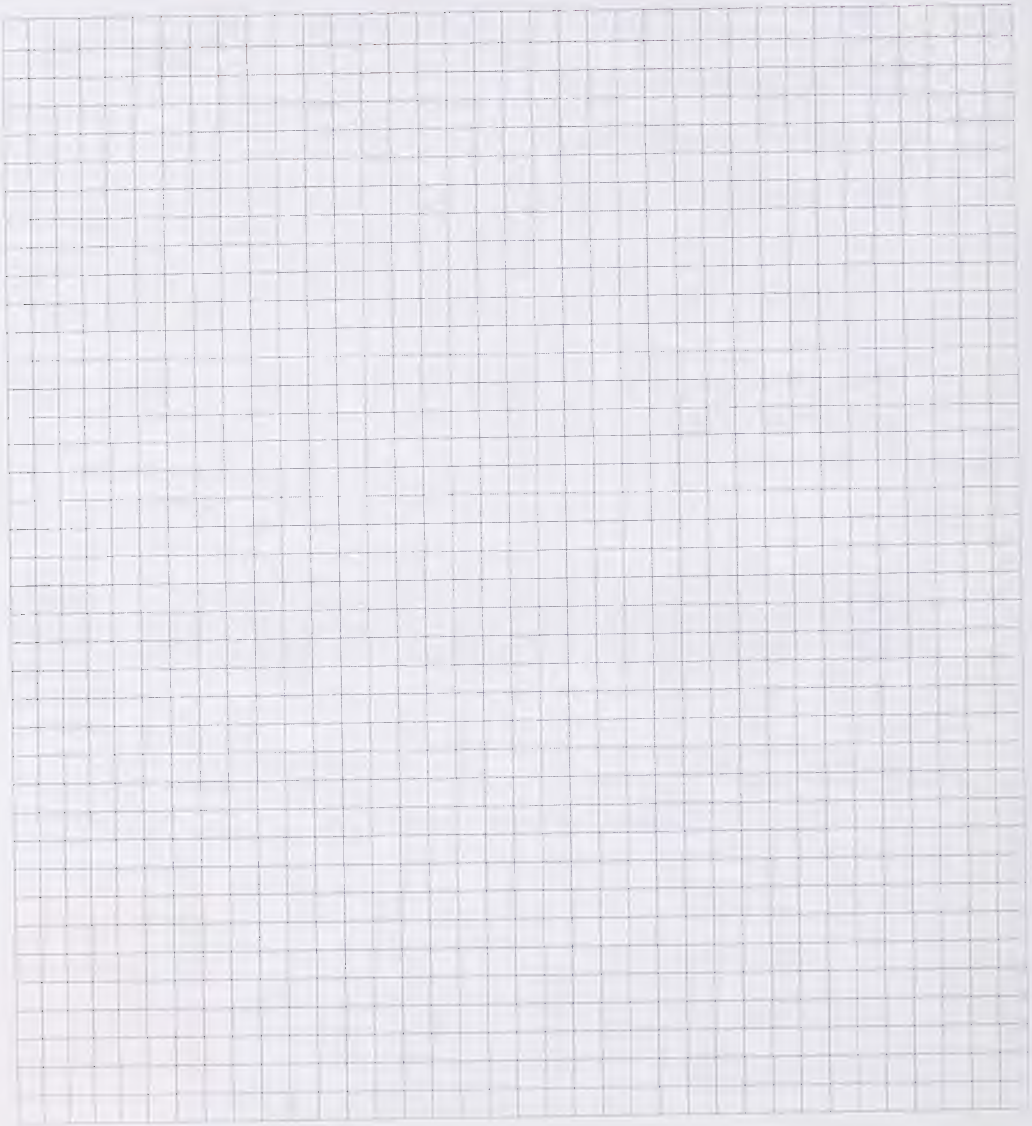


11. Rearrange the Counters and create one more square number.
12. What is the next square number in this pattern? 4, 9, 16, 25, 36, ___
13. Reflect: Does any number have more than one square number as a multiple?

You are now ready to solve problems involving multiples, and to identify the multiples of numbers.

Let's Practice

- Go online to complete the Concept Capsule: Identifying Multiples.
- Turn in your Workbook to Unit 1, Lesson 3 and complete 1 to 16.



Lesson 4

Factors of Whole Numbers

Comparing Ages

Alyssa is twelve years old. She has three cousins. They are Daron, age 2, Rachel age 3, and Matt age 8. Alyssa wrote the following mathematical sentences to show her age:

Reflection

How are the first two examples different from the last?

Daksha is also 12 years old. He has four cousins. They are Anoop, age 2, Bishnu age 4, Dayita, age 5 and Rani, age 6. Can you write mathematical sentences for Daksha's age like those that Alyssa wrote?

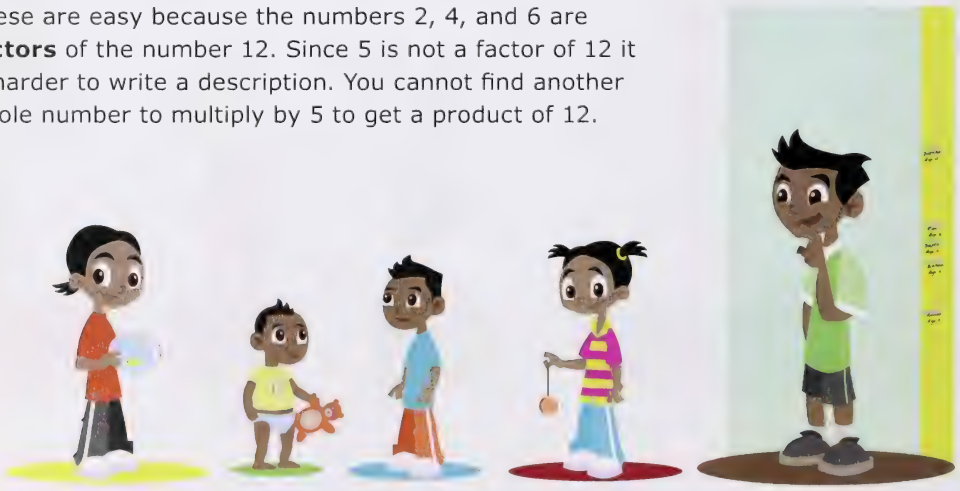
The easy ones should be Anoop, Bishnu and Rani.

Daksha is six times as old as Anoop: $12 = 6 \times 2$

Daksha is three times as old as Bishnu: $12 = 3 \times 4$

Daksha is two times as old as Rani: $12 = 2 \times 6$

These are easy because the numbers 2, 4, and 6 are **factors** of the number 12. Since 5 is not a factor of 12 it is harder to write a description. You cannot find another whole number to multiply by 5 to get a product of 12.



Reflection

Are there any other factors of 12?

Objectives for this Lesson

In this lesson you will explore the following concepts:

- Use arrays to find all the factors of a number
- Explain strategies to find factors of a number
- Use concrete and visual representations to find factors of a number
- Use repeated division to find factors of a number
- Solve problems involving factors or multiples

Factors

A whole number that divides evenly into a number is called a factor of the number. 3 is a factor of 12 because when 3 is multiplied by 4 the product is 12.

$$\underbrace{12}_{\text{Product}} = \underbrace{3 \times 4}_{\text{Factors}}$$

4 is also a factor of 12 because when 4 is multiplied by 3 the product is 12.

Example 1

Identify the factors in the following number sentences:

a. 12 x 7 = 84 b. 15 x 3 = 45 c. 76 x 4 = 304

a. The factors of 84 in this number sentence are 12 and 7.

b. The factors of 45 in this number sentence are 15 and 3.

c. The factors of 304 in this number sentence are 76 and 4.

In some cases you can use your knowledge of multiplication facts to 12 x 12 to find the factors of a number. You may also need to use some methods when the numbers are a little harder.

Example 2

Identify the factors of 24.

Start with 1 and list multiplication sentences that have a product of 24:

1 x 24 = 24 2 x 12 = 24 3 x 8 = 24 4 x 6 = 24

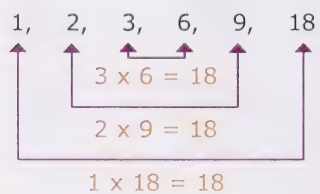
At some point the factors will reverse and you will be listing the same factors all over again:

6 x 4 = 24 8 x 3 = 24 12 x 2 = 24 1 x 24 = 24

The factors of 24 are: 1, 2, 3, 4, 6, 8, 12, 24

You may also notice a pattern to listing the factors of a number:

Factors of 18:



Sometimes, this type of pattern can help you list your factors.

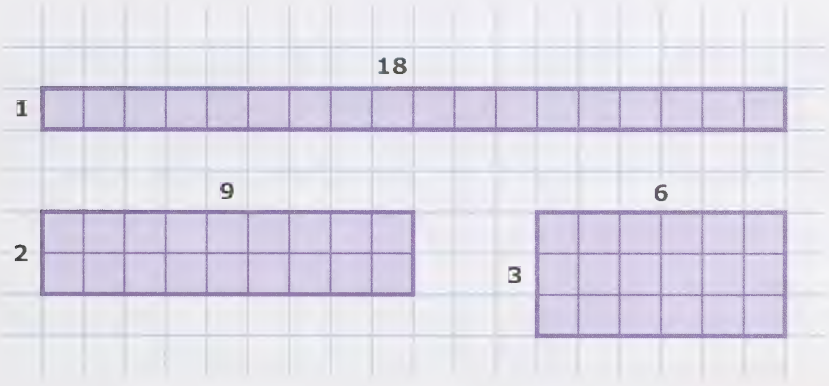
Using Arrays to Find Factors

Just as you used arrays to find the multiples of a number you can also use them to find factors of a number.

Example 3

Use the grid paper to find all of the arrays that are made using a total of 18 squares. List the factors of 18.

You need to make as many unique rectangles as possible that have a total of 18 squares:



When you cannot find any more unique rectangles for the number of squares needed, you have found all the factors of the number.

The factors of 18 are: 1, 2, 3, 6, 9, 18

Some numbers have more factors and some have fewer factors than 18.

Reflection

Do you think the size of the number determines the number of factors? Why or why not?

Let's Explore

Exploration 1: Arrays to Factors

Materials: Unit 1, Lesson 4, Exploration 1 page in your Workbook, Grid Paper from the back of this Unit in your Workbook (you may need two pieces), 5 Pencil Crayons, Pencil

For 1 – 5: Use a different coloured pencil crayon for each.

1. Make all possible arrays of 36 squares on your grid paper. List the factors of 36.
2. Make all possible arrays of 21 squares on your grid paper. List the factors of 21.
3. Make all possible arrays of 32 squares on your grid paper. List the factors of 32.
4. Make all possible arrays of 17 squares on your grid paper. List the factors of 17.
5. Make all possible arrays of 29 squares on your grid paper. List the factors of 29.

The proper factors of a number are all of the factors less than the number but greater than 1. The proper factors of 18 are: 2, 3, 6, 9

6. How many proper factors does the number 29 have?

7. Reflect: What is different about the factors of 29 compared to the factors of 32?

8. Reflect: How many numbers from 1 to 20 have no proper factors?

Using Repeated Division to Find Factors

You can also use division to identify factors of a number.

The factors of 18 using division:

$18 \div 1 = 18$ $18 \div 2 = 9$ $18 \div 3 = 6$ $18 \div 6 = 3$ $18 \div 9 = 2$

To start finding factors of larger numbers you may want to use division. A number is always divisible by 1 and itself. To find the factors you may want to start with a number larger than 1.

Example 4

Zach has 88 baseball cards. He wants to arrange them in rows with an equal number of cards in each row on his wall. What are the different arrangements can he make for the cards?

You need to know all of the multiplication sentences for a product of 88. You can use upside down repeated division to find the factors of 88. Only divide by numbers that are divisible by 1 and itself.

1. Divide by a number and write the division upside down like this:

$2\overline{)88}$

2. Now divide the 2 into 88 and write the answer beneath the 88:

$$2\overline{)88}$$
$$44$$

3. Repeat the process going down the page until you cannot divide anymore:

$$2\overline{)88}$$
$$\overline{)44}$$
$$22$$

$$2\overline{)88}$$
$$\overline{)44}$$
$$\overline{)22}$$
$$11$$

The only whole numbers that divide 11 evenly are 1 and itself, so you have finished dividing.

4. List the factors that you found:

   1, 2, 11, 22, 44, 88

5. List all number sentences:
   You have the following number sentences that are complete:

   1 x 88 = 88

   2 x 44 = 88

   And you have the following that are incomplete:

   __ x 11 = 88

   __ x 22 = 88

   Complete these sentences:

   8 x 11 = 88

   4 x 22 = 88

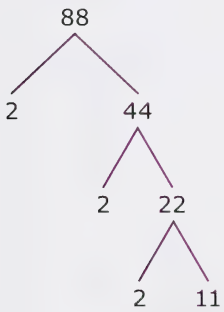
6. Answer the question:

Zach can make the following arrangements:

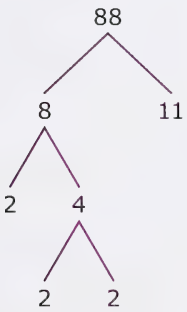
1 row of 88 cards	2 rows of 44 cards
4 rows of 22 cards	8 rows of 11 cards
88 rows of 1 card	44 rows of 2 cards
22 rows of 4 cards	11 rows of 8 cards

You can also use **factor trees** to track the numbers that you have divided into your number.

One Factor tree of 88:



Another Factor tree of 88:



Both methods allow you to find several factors of 88. You can then use them to identify the rest:

1, 2, _____, _____, 11, 22, 44, 88

? X 11 = 88

? x 22 = 88

1, 2, 4, 8, 11, 22, 44, 88

1, 2, 4, 8, 11, _____, _____, 88

4 x ? = 88

2 x ? = 88

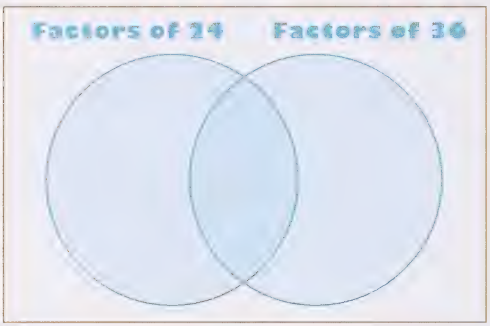
1, 2, 4, 8, 11, 22, 44, 88

Comparing Factors

You can use **Venn diagrams** to show the relationship between factors of two numbers.

Example 5

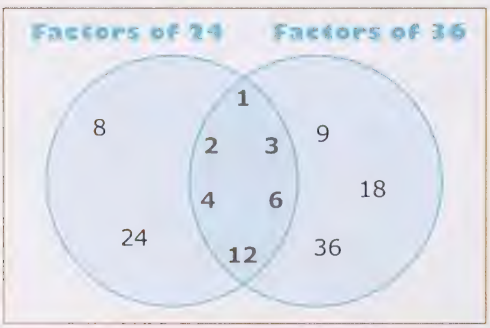
Complete the Venn diagram with all possible factors of each number.



List the factors of each number.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

The factors they have in common go in the intersection of the circles.



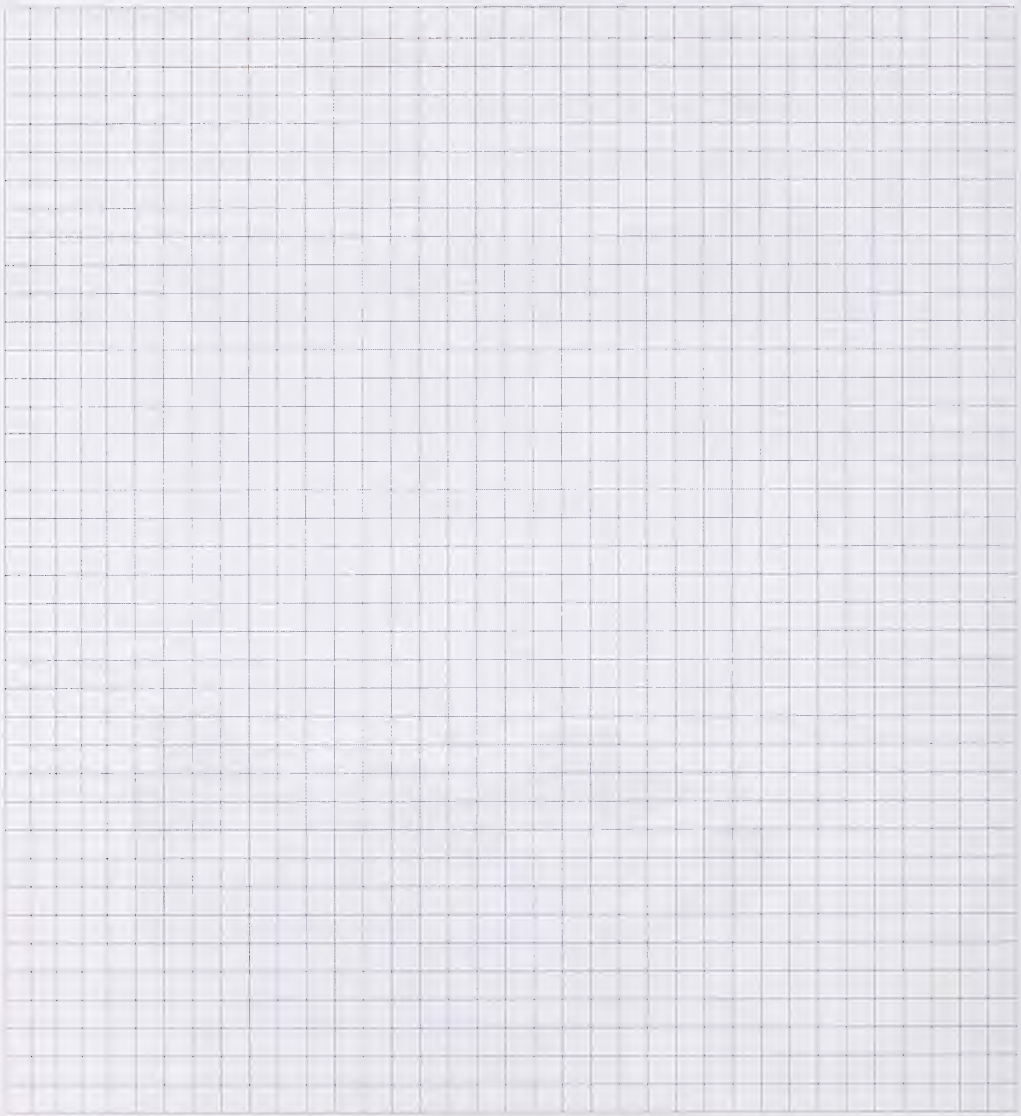
You can use your Venn diagram to answer questions like:

- Do the numbers share any factors?
- What is the largest factor these two numbers share?
- How many factors do the numbers share?
- Which number has more factors?

Let's Practice

- Go online to watch the Notepad Tutor Lesson: Comparing Factors Using Graphic Organizers.

- Turn in your Workbook to Unit 1, Lesson 4 and complete 1 to 21.



Lesson 5

Prime and Composite Numbers

Numbers in Nature

Did you know that the number of petals on a flower is almost always from a special set of numbers? Most often, the number of petals on a flower is one of these numbers:

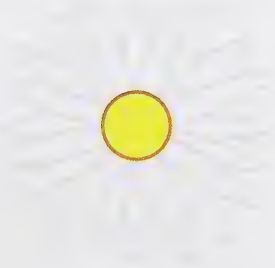
3, 5, 8, 13, 21, 34 or 55

Here are a few specific examples:

Flower	Number of Petals
Lily	3
Buttercups	5
Chicory	21
Daisy	21, 34 or 55

Black-eyed Susan with 13 Petals

Shasta Daisy with 21 Petals



These are special numbers from a pattern called the Fibonacci sequence.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55...

There are two different types of numbers in this sequence. Some of them have only two factors: 1 and itself. Some have more factors.

Leonardo da Pisa, who lived in Italy around the year 1200 was known as Fibonacci. He discovered this special sequence of numbers that often appear in nature.

Leonardo da Pisa, "Fibonacci"

Reflection

Which numbers of the Fibonacci sequence have factors that are not limited to 1 and itself?

Objectives for this Lesson

In this lesson you will explore the following concepts:

- Provide an example of a prime number
- Provide an example of a composite number
- Sort a given set of numbers as prime and composite.

Prime and Composite Numbers

You can use what you know about factors of numbers to classify whole numbers into two categories: **prime numbers**, and **composite numbers**.

Term	Definition	Example
Prime Number	A counting number greater than 1 that has exactly two different whole number factors, 1 and itself.	5 = 1 x 5, and 5 has no other whole number factors
Composite Number	A counting number with more than two different whole number factors greater than 1.	8 = 1 x 8 8 = 2 x 4 The factors of 8 are 1, 2, 4 and 8.

You can use these definitions to classify any whole number.

Example 1

Write all of the factors for each number. Next, identify the number as prime or composite.

a. 12 b. 17 c. 36 d. 41

	Factors	Prime or Composite?
a.	1, 2, 3, 4, 6, 12	Composite
b.	1, 17	Prime
c.	1, 2, 3, 4, 6, 9, 12, 18, 36	Composite
d.	1, 41	Prime

In order for a number to be prime, it can only be written as a product of two unique factors.

- ONE of the two factors is 1.
- ONE of the two factors is a whole number, but is NOT 0 or 1.

Factor Not 1 or 0	×	Factor must be 1	=	Product

You can find factors easily enough if you know your multiplication facts. It is a bit harder to think backward when identifying the prime numbers.

Use the following Exploration to identify all of the prime numbers up to 100.

Let's Explore

Exploration 1: Can You Find the Primes?

Materials: Unit 1, Lesson 5, Exploration 1 page in your Workbook, A Pencil Crayon, Pencil

For 1 – 5: Use the Number Chart on the Exploration page in your Workbook.

1. Colour the 1 and circle the first four prime numbers: 2, 3, 5, 7
2. Colour the multiples of 2 but do not colour the 2.
3. Colour the multiples of 3 but do not colour the 3.
4. Colour the multiples of 5 but do not colour the 5.
5. Colour the multiples of 7 but do not colour the 7.
6. List the numbers that are circled and the numbers that are not coloured in order.
7. These are the prime numbers up to 100. Check each number to be sure that it has no factors other than 1 and itself.
8. Reflect: Why do you think you did not have to colour multiples of 6?
9. Reflect: Why do you think you did not have to colour multiples of 11?
10. What is the smallest prime number greater than 100? Describe how you found that number.

Let's Practice

- Turn in your Workbook to Unit 1, Lesson 5 and complete 1 to 18.

Prime Number Factors

For each composite number greater than one you can find a unique set of two or more prime numbers that are factors of the number. These primes will give you a product of the number. This is called the **prime factorization** of a number.

Here is the prime factorization of each composite number up to 20.

Number	Prime Factorization
4	2 x 2
6	2 x 3
8	2 x 2 x 2
9	3 x 3
10	2 x 5
12	2 x 2 x 3
14	2 x 7
15	3 x 5
16	2 x 2 x 2 x 2
18	2 x 3 x 3
20	2 x 2 x 5

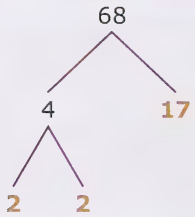
To find the prime factorization of a number you can use a factor tree. This is a way of reducing the number down to the primes, by using repeated division.

Example 2

Find the prime factorization of 68.

1. Start by dividing by any number. Here you see how to start with dividing by 4:

2. Now look at the factors you have so far. Can you find any factors of 4? If so, divide again



Now look at 17. Since 17 is prime and 2 is also prime you are finished.

The prime factorization of 68 = 2 x 2 x 17

Example 3

Which number has a prime factorization of 2 x 3 x 3 x 5?

Prime Factorization: → 2 x 3 x 3 x 5

Multiply the numbers in order from left to right: → 6 x 3 x 5

Now multiply 18 x 5: → 18 x 5

Finally, the answer: → 90

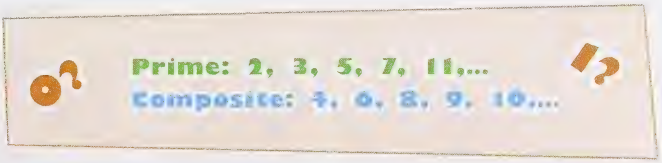
- Go online to complete the Concept Capsule: Prime Factorization.

Let's Practice

- Turn in your Workbook to Unit 1, Lesson 5 and complete 19 to 26.

What About 1 and 0?

The definition of a prime number leaves out 1 and 0. These are also whole numbers. Why do you think they are not included in the prime number set?



The definition tells you that a prime number is divisible by 1 and the number itself. It also says that they must be two different numbers.

The number 1 can be expressed as a product in many ways:

1 x 1
1 x 1 x 1
1 x 1 x 1 x 1
...and so on.

This means that by the definition of prime numbers, 1 cannot be a prime.

The number 0 can also be expressed in many ways:

0 x 1
0 x 2
0 x 3
...and so on.

By studying the definition you can see that 0 and 1 are not prime numbers.

Since they are not considered prime, you may think, "Why are 0 and 1 not composite numbers?"

A composite number must be the product of two factors that are

- not the number itself.
- different from 1 and the number.

Look at 1 and 0:

1 only has one factor, not two different factors: 1 x 1 = 1

0 is not a product of two factors that are not 0. 0 is always a product of one or more factors that include 0.

You should now be able to explain why 1 and 0 are not prime or composite.

Let's Practice

- Turn in your Workbook to Unit 1, Lesson 5 and complete 27 to 28.

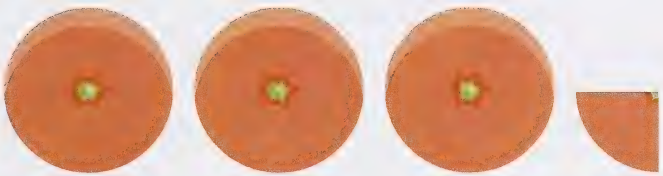
Lesson 6

Modelling Fractions

Describing Oranges

Canada is the largest importer of oranges grown in the United States. To grow oranges there needs to be a long season of warm weather. Freezing temperatures can kill the fruit and even the trees that produce the oranges.

Lian has three full oranges and one-quarter of an orange.



The number of oranges can be expressed as a mixed number: $3\frac{1}{4}$

3 represents the whole oranges and $\frac{1}{4}$ represents the part of the last orange.

The number of oranges can also be expressed as the number of quarters. Lian shows this by dividing the 3 oranges that are whole into quarters:

There are 13 quarter oranges or $\frac{13}{4}$.

Reflection

If you have four and a half oranges how many halves do you have?

Objectives for this Lesson

In this lesson you will explore the following concepts:

- Model mixed numbers and improper fractions
- Translate a given improper fraction between concrete, pictorial and symbolic forms
- Translate a given mixed number between concrete, pictorial and symbolic forms

- Go online to complete the Concept Capsule: Fractions – Parts of a Set.

Improper Fractions

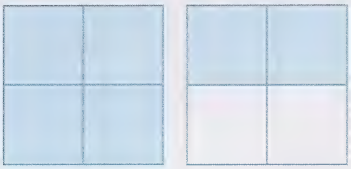
A **proper fraction** has a **denominator** that is greater than the **numerator**. This means that the value of the fraction is between 0 and 1.

Proper Fraction

$\frac{3}{4}$

An **improper fraction** has a denominator that is less than the numerator. It represents a value of more than one.

Improper Fraction



$\frac{6}{4}$

When you write an improper fraction representing a model or a picture you should begin by counting the number of parts. In this case each square is divided into four parts. The 4 becomes the denominator. The numerator is found by counting the shaded parts. In this case there are 6 shaded parts.

Example 1

Nina has four and a half apples. She cuts her apples into pieces that are each $\frac{1}{4}$ of an apple. What improper fraction represents her apples?

A picture for the model would be:

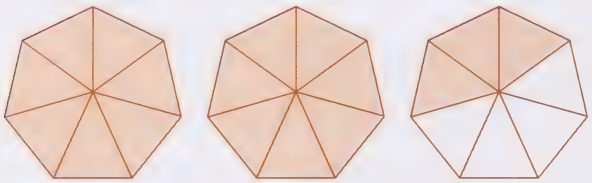
There are 18 shaded parts. The numerator will be 18.

Each apple is in 4 pieces. The denominator will be 4.

Nina has $\frac{18}{4}$ apples.

Example 2

Write an improper fraction for the given figure.



There are 7 parts in each figure. The denominator will be 7.

17 parts are shaded in the three figures. The numerator will be 17.

The improper fraction is $\frac{17}{7}$.

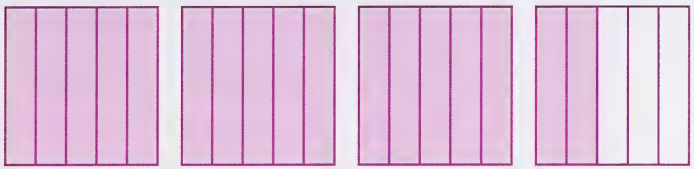
Mixed Numbers

Mixed numbers are a combination of whole numbers and proper fractions. They are used to express fractions greater than 1.

The whole number 2 represents the two full shaded squares. The denominator of the fraction represents the 4 parts of the squares and the numerator 1 represents the 1 part of the last square that is shaded.

Example 3

Write a mixed number for the given figure.



There are 3 whole rectangles shaded. The whole number is 3.

There are 5 parts to each rectangle. The denominator is 5.

There are 2 parts shaded in the last rectangle. The numerator is 2.

The mixed number is $3\frac{2}{5}$.

Fractions Greater than 1

You should have noticed that both improper fractions and mixed numbers represent fractions greater than 1. A mixed number or an improper fraction can both represent the same number. Use the Exploration to express models as both types.

Let's Explore

Exploration 1: Pattern Block Fractions

Materials: Unit 1, Lesson 6 Exploration page in your Workbook, Pattern Blocks from the back of this Unit in your Workbook, Pencil, Scissors

1. Cut out the Pattern Blocks.
2. Take six triangles and manipulate them to cover 1 hexagon. Use more of the triangles and hexagons to create a model for the improper fraction $\frac{14}{6}$.

3. Write a mixed number for $\frac{14}{6}$.
4. Take three triangles and manipulate them to cover 1 trapezoid. Use more of the triangles and trapezoids to create a model for the mixed number $2\frac{1}{3}$.
5. Write an improper fraction for $2\frac{1}{3}$.
6. Create your own model for an improper fraction. Write the improper fraction and the mixed number for your model.
7. Create your own model for a mixed number. Write the improper fraction and the mixed number for your model.
8. Daksha has $4\frac{1}{3}$ granola bars. Create a model for Daksha's granola bars.
9. Using these pattern blocks, what are some other fractions you can model that are greater than 1? Sketch a model of your fractions.
10. Reflect: What are some improper fractions you cannot model using these pattern blocks?
11. Reflect: What are some mixed numbers you cannot model using these pattern blocks?

You can also use counters to represent whole numbers and parts of whole numbers. Gather at least 40 counters, such as dry beans, to use with the following example.

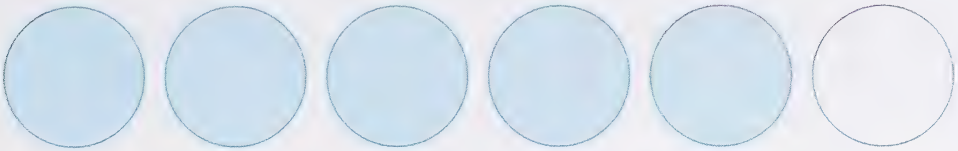
Example 4

Nina has $5\frac{1}{6}$ tangerines. Create a model for the number of tangerines that Nina has. What is the number of tangerines, written as an improper fraction?

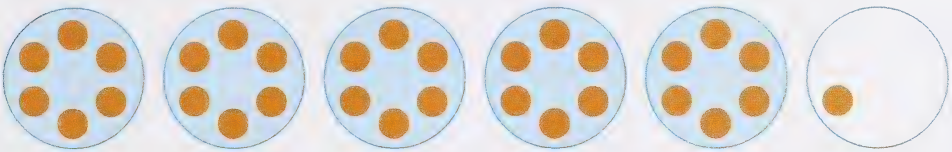
Draw one more circle on your paper than the whole number in the problem:

Nina has $5\frac{1}{6}$ tangerines so 5 is the whole number. Draw 6 circles:

Use your pencil crayons to shade the five wholes:



Place 6 counters in each whole circle to represent the six parts in each. Place 1 counter in the last circle to represent the one-sixth of a tangerine:



You have now created a model for $5\frac{1}{6}$. You can count the counters to find out how many parts are in the number and write the improper fraction.

There are 31 parts. The numerator is 31.

There are 6 parts in each. The denominator is 6.

The improper fraction is: $\frac{31}{6}$

You can also use counters to create models of improper fractions. Use counters as you follow along with the next example.

Example 5

Cameron has $\frac{13}{3}$ sticks of sugar-free chewing gum. Create a model for the number of sticks of gum Cameron has and write the number as a mixed number.

Gather 13 counters:

Group the counters into groups of 3 and draw one circle around each group. Also draw a circle around the extras:

Shade the circles that have 3 counters:

There are 4 wholes, and 1 part in the final circle.

The mixed number is $4\frac{1}{3}$.

Let's Practice

- Turn in your Workbook to Unit 1, Lesson 6 and complete 1 to 15.
- Go online to complete the Concept Capsule: Adding Mixed Numbers with Like Denominators.

Lesson 7

Fraction Relationships

Water Conservation

Each person in Canada uses an average of 1 600 cubic metres of water per year. Canada consumes the fourth largest amount of water out of 29 countries tracked. Water is a resource that should be conserved through proper use. There are appliances in your home that use water. Changing the settings on these appliances can save water.

Lian wants to know how much water her family uses doing laundry. She reads her washing machine manual to find out how much water is used in each full load. She tracks the number of loads of laundry her family does in a week.

Number of Wash Loads	1	2	3	4	5	6
Litres of Water	$132\frac{1}{2}$	265	$397\frac{1}{2}$	530	$662\frac{1}{2}$	795

Reflection

What do you notice about the pattern in the table? How can you use the pattern to find the amount of water used for a week? A year?

Objectives for this Lesson

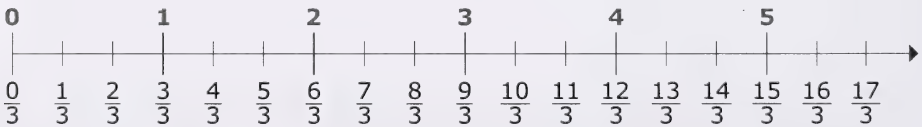
In this lesson you will explore the following concepts:

- Place a given set of fractions on a number line
- Predict the value of an unknown term in a table of values
- Express improper fractions as mixed numbers
- Express mixed numbers as improper fractions

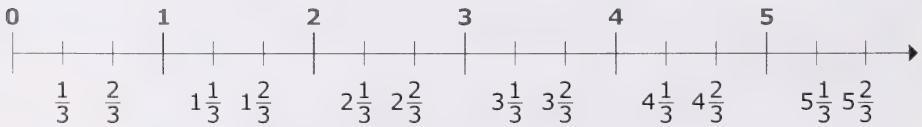
Fractions on a Number Line

You can use a number line to identify improper fractions and mixed numbers. The number of parts between the whole numbers let you know the denominator.

With this number line you can see that there are three parts between each pair of whole numbers. This means that the fractions will represent thirds, and the denominators will be 3.



You can also use the number line to identify mixed numbers between the whole numbers:



Now you need to know how to plot fractions on a number line. The following examples will help you with this concept.

Example 1

Write the improper fraction and the mixed number for each letter on the number line.

The number line is divided into 5 parts. Each denominator will be 5.

A. The **A** is on the sixth mark after the 0. That means the numerator is 6.

Improper fraction: $\frac{6}{5}$

The letter A is after the mark for the number 1 and before the mark for the number 2.

That means that the whole number is 1. It is on the first mark after 1 so the numerator is 1.

Mixed number: $1\frac{1}{5}$

B. The **B** is on the twelfth mark after the 0 so the numerator is 12.

Improper fraction: $\frac{12}{5}$

The letter B is after the mark for the number 2 and before the mark for the number 3.

That means the whole number is 2. It is on the second mark after 2 so the numerator is 2.

Mixed number: $2\frac{2}{5}$

C. The **C** is on the 19th mark after 0 so the numerator is 19.

Improper fraction: $\frac{19}{5}$

The letter C is after the mark for the number 3 mark and before the mark for the number 4.

That means the whole number is 3. It is on the fourth mark after the 3 so the numerator is 4.

Mixed number: $3\frac{4}{5}$

Let's Explore

Exploration 1: Fractional Number Lines

Materials: Unit 1, Lesson 7, Exploration 1 page in your Workbook, Blank Number Lines from the back of this Unit in your Workbook, Pencil

1. Create a number line that starts at 4 and has five parts between each whole number. Label the marks with the correct mixed numbers.
2. Place a point at $4\frac{2}{5}$ and label it A.
3. Place a point at $4\frac{4}{5}$ and label it B.
4. Place a point at $5\frac{1}{5}$ and label it C.
5. Create a number line that starts with 3 and has six parts between each whole number. Label the marks with the correct improper fractions.
6. Place a point at $\frac{21}{6}$ and label it A.
7. Place a point at $\frac{26}{6}$ and label it B.
8. Place a point at $\frac{29}{6}$ and label it C.
9. Create a number line that is able to contain the following points: A $2\frac{1}{4}$, B $3\frac{2}{4}$, and C $4\frac{3}{4}$. Label the line with those points.
10. Create a number line that is able to contain the following points: A $\frac{25}{7}$, B $\frac{30}{7}$, and C $\frac{34}{7}$. Label the line with those points.

- Go online to watch the Notepad Tutor Lesson: Making Predictions.

Patterns in Tables

Using the pattern in a table of values can help you see the relationship between improper fractions and mixed numbers.

This table shows a set of improper fractions and the mixed numbers that they are equal to:

Improper Fraction	$\frac{4}{3}$	$\frac{5}{3}$	$\frac{6}{3}$	$\frac{7}{3}$	$\frac{8}{3}$	$\frac{9}{3}$	$\frac{10}{3}$	$\frac{11}{3}$
Whole Number or Mixed Number	$1\frac{1}{3}$	$1\frac{2}{3}$	2	$2\frac{1}{3}$	$2\frac{2}{3}$	3	$3\frac{1}{3}$	$3\frac{2}{3}$

In the improper fraction row, the whole numbers are expressed as fractions. That allows you to see the pattern to counting by thirds.

Reflection

Can you predict the next three improper fractions in the pattern?

The patterns in mixed numbers and improper fractions can help you find missing values in a table. You can also use fractions to extend a pattern.

Example 2

Nina and Cameron are adding orange pieces to a bowl. The table shows how the orange pieces have been added. What numbers are missing from the table?

Number of times part of an orange is added	3	4	5	6	7	8	9	10
Total oranges added	$\frac{3}{4}$	1		$1\frac{2}{4}$		2	$2\frac{1}{4}$	

The number of orange pieces added the 3rd time is a proper fraction. The mixed numbers show that this pattern should follow a pattern of mixed numbers. Between 1 and 2 there are two missing numbers. If you are counting by fourths then the missing number to the left of $1\frac{2}{4}$ would be one less in the numerator. The missing number to the right would be one more:

Number of times part of an orange is added	3	4	5	6	7	8	9	10
Total oranges added	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{2}{4}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	

Follow the same reasoning to find the mixed number to the right of $2\frac{1}{4}$:

Number of times part of an orange is added	3	4	5	6	7	8	9	10
Total oranges added	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{2}{4}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{2}{4}$

When improper fractions are unable to provide an answer to your question, you can convert your improper fraction to a mixed number.

Let's Practice

- Turn in your Workbook to Unit 1, Lesson 7 and complete 1 to 12.

Let's Explore

Exploration 2: Fraction Circles

Materials: Unit 1, Lesson 6, Exploration 2 page in your Workbook, Fraction Circles from the back of this Unit in your Workbook, Pencil, Scissors, Small Plastic Bag

1. Cut out all Fraction Circles and their parts. After completing this Exploration, store the Fraction Circles in the small plastic bag. You will use them again later.
2. Create a model for each of the following:

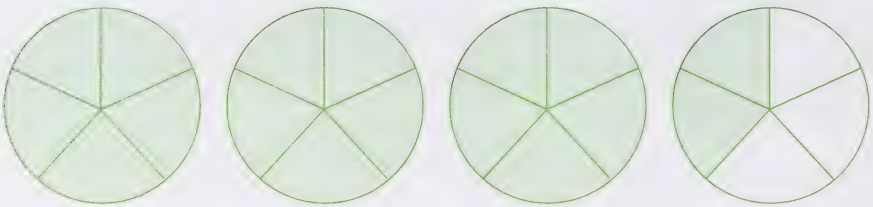
   a. $\frac{12}{5}$ b. $2\frac{2}{5}$
3. What do the models have in common? Why?
4. Write an improper fraction that you can model using the circles that have 7 pieces. Model that fraction and draw your model.
5. Write a mixed number that you can model using the circles that have 8 pieces. Model that fraction and draw your model.
6. Cameron used 4 of the circles that have 4 parts each to model $\frac{17}{5}$. What was his mistake?
7. Lian needs to model the fraction $4\frac{1}{6}$. Describe the parts she needs to use to model this fraction.

8. Reflect: Write three improper fractions that you cannot model using these circles and their parts.

9. Reflect: Write three mixed numbers that you cannot model using these circles and their parts.

Converting Fractions

Here is how the improper fraction relates to the mixed number.



Improper Fraction: $\frac{17}{5}$ Mixed Number: $3\frac{2}{5}$

You can see that the denominator stays the same for both fractions. You can convert improper fractions to mixed numbers using a process.

If you know the improper fraction and need to find the mixed number you can use division:

Divide the numerator by the denominator:

$$\begin{array}{r} 3 \\ 5\overline{)17} \\ -\,15 \\ \hline 2 \end{array}$$

3 ← Number of Wholes

2 ← Number of Fifths

This 5 divides into 17 a total of 3 times with a remainder of 2.
You can use this to write the mixed number:

5 divides into 17 a total of **3** times. → $3\frac{2}{5}$ ← The remainder is **2**.

The denominator stays **5**.

Example 3

Write the improper fraction $\frac{36}{7}$ as a mixed number.

Divide 7 into 36:

$$\begin{array}{r} 5 \\ 7\overline{)36} \\ -\,35 \\ \hline 1 \end{array}$$

The whole number is 5, the numerator is the remainder 1 and the denominator is 7.

The mixed number is: $5\frac{1}{7}$

If you know the mixed number and need to find the improper fraction you can use another process.

To write $3\frac{2}{5}$ as an improper fraction:

Multiply the denominator by the whole number:

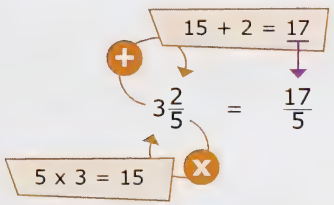
$3\frac{2}{5}$ $\quad 5 \times 3 = 15$

Add the numerator:

$15 + 2 = 17$

This is the numerator of your improper fraction: $\frac{17}{5}$

You can also look at it this way:



To get the numerator of your improper fraction: Multiply the denominator by the whole number and add the numerator to that answer.

Example 4

Write the improper fraction for $6\frac{2}{13}$.

Multiply the denominator by the whole number:

$6\frac{2}{13}$ $13 \times 6 = 78$

Add 2 to the answer of 78:

$6\frac{2}{13}$ $78 + 2 = 80$

Write the improper fraction:

$\frac{80}{13}$

You are now ready to convert fractions!

Let's Practice

- Turn in your Workbook to Unit 1, Lesson 7 and complete 13 to 25.

Lesson 8

Finding Patterns in Tables

Nutrition and Health

The amount of fat that you eat contributes to the total number of calories that you consume. Each gram of fat has nine calories. Nutritionists recommend that no more than 30% of any person's daily calories come from fat. The chart below provides you with the maximum amount of fat you should eat based on the number of calories consumed.

Calories	500	750	1 000	1 250	1 500	1 750
Number of Fat Grams	$16\frac{2}{3}$	25	$33\frac{1}{3}$	$41\frac{2}{3}$	50	$58\frac{1}{3}$

Reflection

Did you see a pattern in the "Number of Fat Grams" portion of that table? How would the pattern change if the number of calories changed to a different increment?

Tables can help you solve problems, or predict future **terms** of a pattern. You will be able to solve problems using tables of values after this lesson.

Objectives for this Lesson

In this lesson you will explore the following concepts:

- Create a pictorial representation of the relationship shown in a table of values
- Identify missing elements in a given table of values
- Identify errors in a given table of values
- Describe the pattern within each column of a given table of values

Modelling Table Patterns

You can create a picture to represent the values in a table to help you see the relationship between the numbers. You may want to use grid paper or simply use dots or other figures that are easy to draw.

Example 1

Create a model for the pattern in the table:

People	Time to Front of Line (min)
5	$1\frac{1}{4}$
6	$1\frac{2}{4}$
7	$1\frac{3}{4}$
8	2
9	$2\frac{1}{4}$
10	$2\frac{2}{4}$
11	$2\frac{3}{4}$
12	3
13	$3\frac{1}{4}$

Use a model that is divided into four sections since the fractions for time have denominators of 4. Use a circle to represent time:

To model the first values in the table you need 5 people for 1 and one-fourth minutes:

5 people, $1\frac{1}{4}$ minutes:

6 people, $1\frac{2}{4}$ minutes:

You can continue to model each value in the table. Models can help you visualize the pattern so that you can solve a problem.

When modelling decimals you may want to use grid paper or Base 10 Blocks.

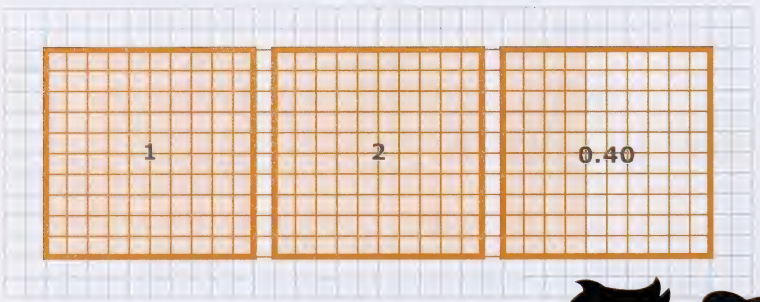
Example 2

Create a model for the pattern in the table:

Hours	1	2	3	4	5	6	7	8
Dollars Saved	2.40	2.80	3.20	3.60	4.00	4.40	4.80	5.20

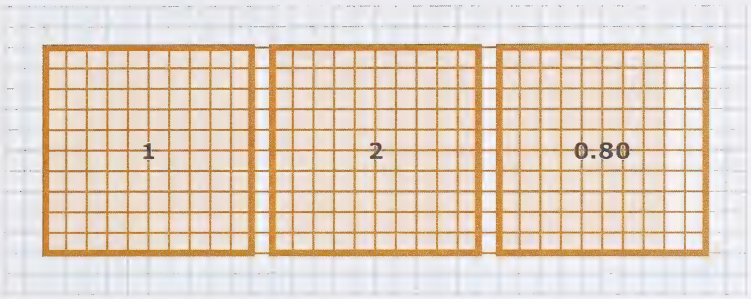
Using grid paper you can model terms in the pattern like this:

Hour 1:

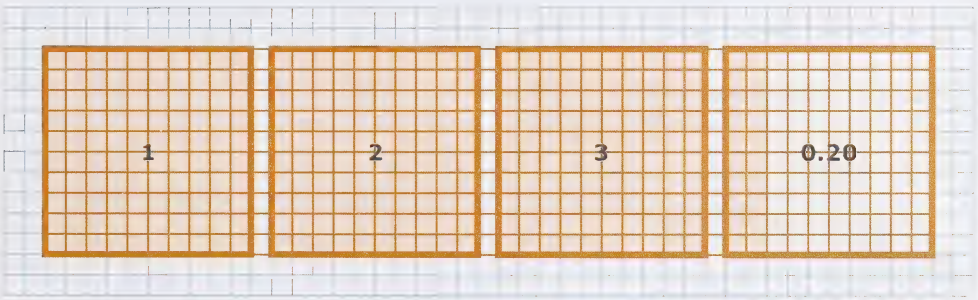


Continue to use the grid paper to model the numbers in the table for dollars saved. This is to help you find the pattern.

Hour 2:



Hour 3:



By now, you may have noticed the pattern of "add 0.40". If not, continue to model the numbers until you do.

Describing Patterns

To describe a pattern you should use mathematical language.
There are two elements that must be present:

1. A description of the operation (add, subtract, multiply or divide)
2. A value to describe the change between terms in the pattern

Example 3

Describe the pattern in the table.

Hours	1	2	3	4	5	6	7	8
Dollars Saved	2.40	2.80	3.20	3.60	4.00	4.40	4.80	5.20

Earlier, you made a model for this problem. You could use your model to discover the operation and the value that describe your pattern.

This pattern grows from unit to unit. It must be addition or multiplication. By looking at the first two terms and then the next two, decide if there is a number you can add to go from one to the next each time:

Hours	1	2	3	4	5	6	7	8
Dollars Saved	2.40	2.80	3.20	3.60	4.00	4.40	4.80	5.20

+0.40 +0.40

Now check this between each pair of terms to make sure:

Hours	1	2	3	4	5	6	7	8
Dollars Saved	2.40	2.80	3.20	3.60	4.00	4.40	4.80	5.20
		+0.40	+0.40	+0.40	+0.4	+0.40	+0.40	+0.40

Since this fits the pattern, you are ready to describe it in words:

The pattern is "**add 0.40**".

- Go online to complete the Concept Capsule: Modelling and Describing Patterns in Tables.

Sometimes, there can be an error in a pattern. Most terms follow the same pattern, but one term is wrong. You can use your model or the reasoning used to describe the pattern to find an error.

Example 4

Nina has been saving the same amount of money every week. Find the error in the pattern of the second column.

Week	Nina's savings (in dollars)
4	48
5	60
6	70
7	84
8	96
9	108

There are several ways you can approach this problem. Here is a simple way.

Find the change between each pair of values in the second column.

Week	Nina's savings (in dollars)	
4	48	
5	60	+ 12
6	70	+ 10
7	84	+ 14
8	96	+ 12
9	108	+ 12

You can see that from week 4 to 5, week 7 to 8 and week 8 to 9 you have the same change. This change is "add 12".

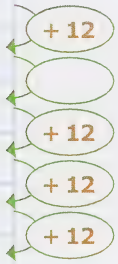
The change from week 5 to 6 is the first that does NOT follow this pattern. Try changing the $70 savings in week 6 by following the pattern "add 12":

Savings in Week 5		+ 12	=	Savings in Week 6
60		+ 12	=	72

Week	Nina's savings (in dollars)
4	48
5	60
6	~~70~~ 72
7	84
8	96
9	108

Now check to see that the change between each pair of terms is consistent:

Week	Nina's savings (in dollars)
4	48
5	60
6	~~70~~ 72
7	84
8	96
9	108



You have found and fixed the error in the table.

Patterns do not always follow a simple operational pattern like "add 5" or "subtract 2.4". You might have to really think to identify some patterns. Use the following Exploration to look for patterns that are different.

Let's Explore

Exploration 1: Pattern Problems

Materials: Unit 1, Lesson 8, Exploration 1 page in your Workbook, Pencil

Find the pattern for each of the following. Identify the next three terms and describe the pattern.

1. M, T, W, T, F, S, S, M...
2. 3.4, 5.4, 10.8, 12.8, 25.6, 27.6, 55.2...
3. 1, 4, 9, 16, 25, 36, 49, 64...

4. 31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31...

5. 7, 171, 272, 373, 474, 575, 676...

Missing Values

When a table of values is missing a number you can find it by observing the pattern. Once you identify the pattern you can fill in any of the missing values.

Example 5

Daksha's family is going to rent a snowmobile during their winter vacation. The chart shows the cost for renting the snowmobile each day.

a. Find the missing values in the table.

b. After 6 days, the rate remains the same for each day. How much will it cost the family to rent the snowmobile for 8 days?

Day Number	1	2	3	4	5	6
Cost of Rental (in dollars)	74.50		49.50	37	24.50	

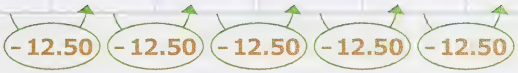
a. Look for a change that is consistent between the **consecutive** terms for 3, 4, and 5 days of rental:

Day Number	1	2	3	4	5	6
Cost of Rental (in dollars)	74.50		49.50	37	24.50	

-12.50 -12.50

Assuming the pattern of "subtract 12.50" is correct, fill in the missing values:

Day Number	1	2	3	4	5	6
Cost of Rental (in dollars)	74.50	62	49.50	37	24.50	12



Since the pattern fits for the missing values you were correct.

b. To find the total cost for 8 days, add together each day's rate.

$74.50 + $62 + $49.50 + $37 + $24.50 + $12 + $12 + $12

It will cost the family $283.50 to rent a snowmobile for 8 days.

Let's Explore

Exploration 2: Body Patterns

Materials: Unit 1, Lesson 8, Exploration 2 page in your Workbook, Pencil

1. Complete the following table with the help of a friend. Add different body parts in last 4 spaces.

Body Part	Number of People							
	1	2	3	4	5	6	7	8
Nose	1							
Eyes	2							
Toes on One Foot	5							
Toes on Two Feet	10							

2. Describe the pattern to the number of toes on one foot.
3. Describe the pattern to the number of toes on two feet.
4. What are some other patterns you found?
5. Reflect: Name something around your home that would have a similar pattern.

Making a Table

It can be helpful to create a table to solve certain problems. You need to decide the parts of the problem. Once that is done, fill in the table with those parts.

Example 6

Alyssa's baseball team has won 6 games and lost 1 game to Cameron's team for each of the past two years. If they continue with this pattern, how many games will they have won after 6 years?

To create a table you need to make a plan.

- The question is "How many games will they win in 6 years?"
- You need two values: number of years and number of wins

Create your table. Make sure the number of years includes the numbers in the question.

Number of Years	1	2	3	4	5	6
Number of Games Won	6	12	18	24	30	36

Now you can tell that the number of games won after 6 years is 36.

Let's Practice

- Turn in your workbook to Unit 1, Lesson 8 and complete 1 to 15.

Lesson 9

Multiplying Decimals

Earning Money

There are many ways that people work to earn an income. Sometimes people do not have a regular job but they find a way to make money to buy the things they need or want. There are people that make crafts or perform services to earn a living.

Reflection

Can you name ways that you have earned money?

Lian is making wooden toys with her dad to sell so she can earn money for a scooter she wants to buy. Lian earns $24.25 for each wooden toy she makes. If she sells 8 wooden toys in one week how much will she earn?

This story problem can be modelled:

This story problem can be expressed as a number sentence:

24.25 x 8 = money earned in one week

To solve the number sentence you must be able to multiply a decimal by a whole number:

24.25 x 8 = 194.00

Lian will earn $194.00 by selling her 8 wooden toys.

Reflection

Can you think of a time when you needed to multiply a dollar amount by a number?

Objectives for this Lesson

In this lesson you will explore the following concepts:

- Predict products of decimals, using estimation strategies
- Place the decimal point in a product
- Solve a given problem that involves multiplication of decimals

Multiplication Review

There are several methods you can use to multiply a two to four digit number by a one digit number. Here are some examples that you may have experienced in prior years.

Distributive Property

You can use the **distributive property** in two ways:

1. Using expanded notation:

   Multiply 453 x 8.

   Write 453 in expanded form: → 400 + 50 + 3
   Multiply each part by 8: → (400 x 8) + (50 x 8) + (3 x 8)
   3 200 + 400 + 24
   Add each product: → 3 624

2. Using traditional columns:

   Multiply 218 x 7.

```
   218
 x   7
 ----
   56   <-- Multiply by the ones place.
  70    <-- Multiply by the tens place.
+1 400  <-- Multiply by the hundreds place
 -----
 1 526  <-- Add.
```

- Go online to complete the Concept Capsule: Understanding the Distributive Property Using Base 10 Blocks.

Multiplication by Regrouping

The regrouping method is the most common:

Multiply 315 x 4.

$\overset{2}{3}15 \times 4 = 0$	1. Multiply 4 x 5. 2. Put the ones digit of the answer below. 3. Put the tens digit of the answer above the tens digit of the problem.	
$\overset{2}{3}15 \times 4 = 60$	1. Multiply 4 x 1 and add 2. 2. Place below the tens digit.	
$\overset{2}{3}15 \times 4 = 1\,260$	1. Multiply 4 x 3 place below the hundreds.	

Now It's Your Turn!

Multiply.

a. 234 x 8

b. 195 x 3

c. 3 427 x 5

Solutions

a. 1 872

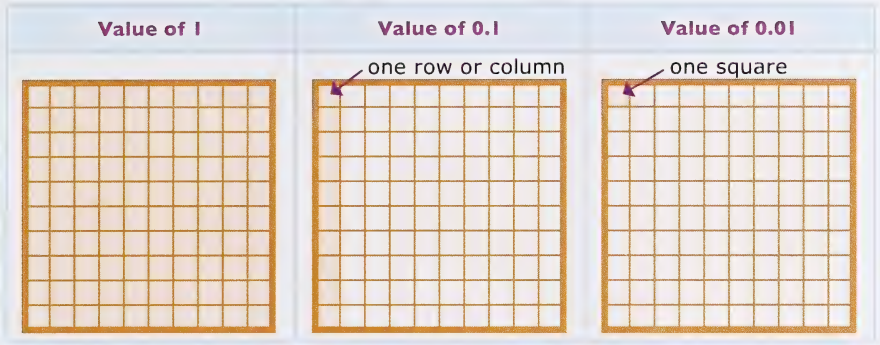
b. 585

c. 17 135

Modelling Decimal Multiplication

You can multiply a decimal by a whole number using the same methods that you used to multiply two whole numbers. The hardest part of multiplying decimals is placing the decimal point in the product. To understand where the decimal is placed it helps to first have an image of the operation.

You can use grid paper to model decimal multiplication. You will use a 10 x 10 grid to model the value of 1. Here are the values of each:

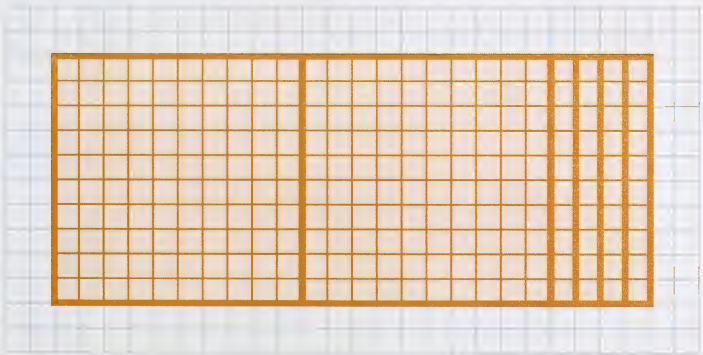


Let's try this together! Use a sheet of Grid Paper to help you with the next problem. You will find Grid Paper at the back of this Unit in your Workbook.

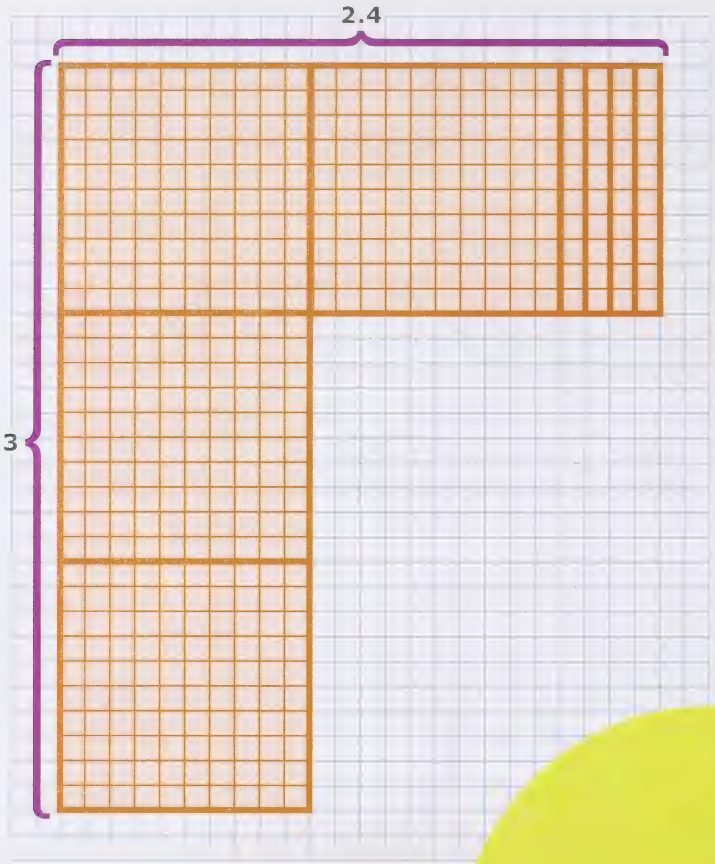
Example 1

Find the product: 2.4 x 3.

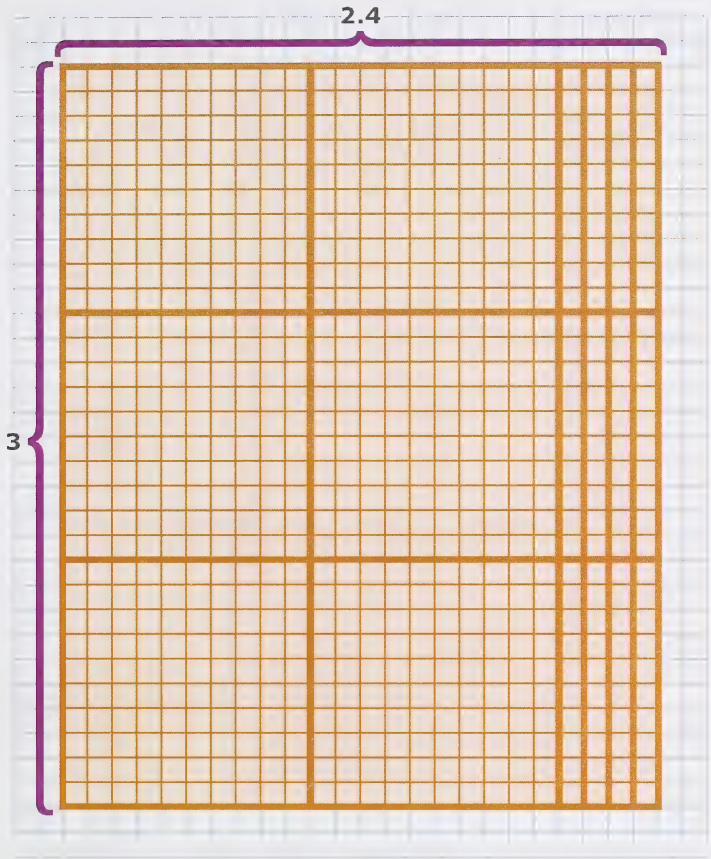
Model 2.4 using grid paper. Place the 10 x 10 squares side by side with no spaces:



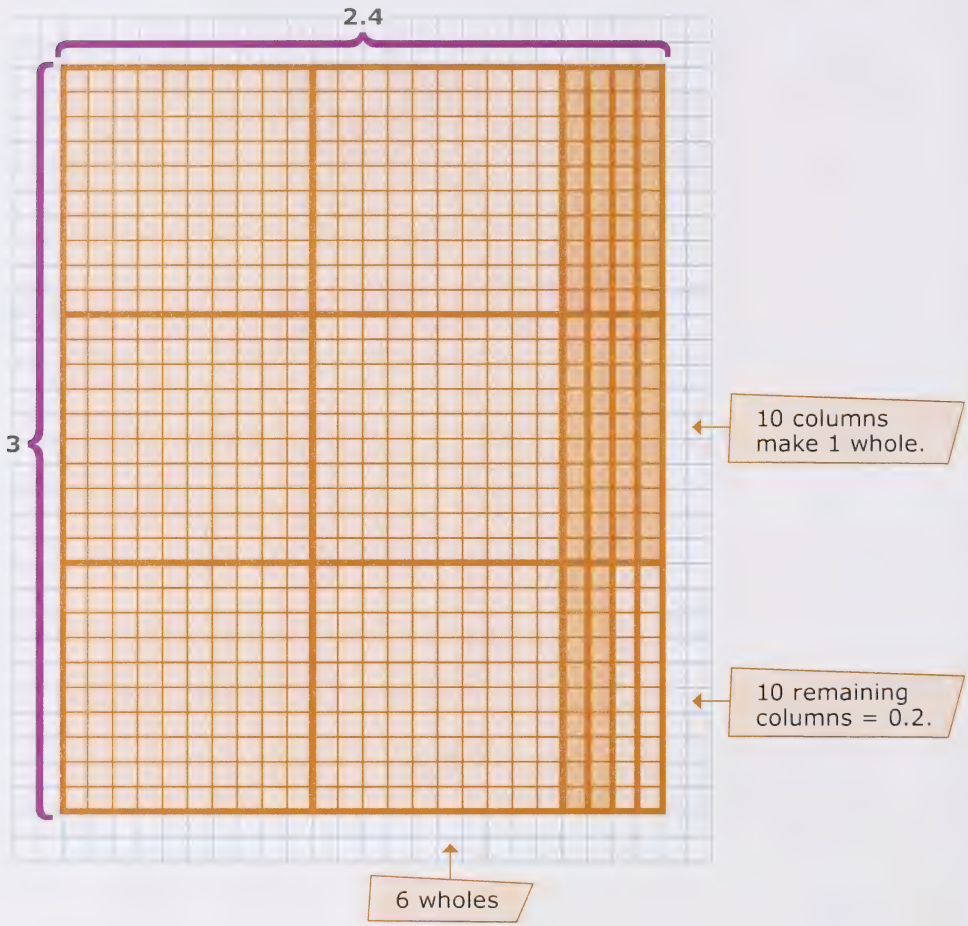
Now model 3 going down from the first square of 2.4:



Now fill in the rest of the rectangle:



Now you need to regroup the tenths to make a one:



Finally you are ready to write the answer:

2.4 x 3 = ?

Based on grid paper model:

$$\begin{array}{r} 6 \\ 1 \\ +\ 0.2 \\ \hline 7.2 \end{array} =$$

Let's Explore

Exploration 1: Modelling Decimal Multiplication

Materials: Unit 1, Lesson 9, Exploration 1 page in your Workbook, Grid Paper from the back of this Unit in your Workbook (2 pages), Pencil

For 1 – 3: Draw your models on Grid Paper.

1. Make a model of 1.8 x 2 and find the solution.
2. Make a model of 3.5 x 3 and find the solution.
3. Create your own multiplication sentence. Make a model of your sentence and solve.

- Go online to watch the Notepad Tutor Lesson: Multiplication with Decimals (1-Digit Whole Number Multiplier).

Placing the Decimal Point

You can solve a multiplication sentence that has a decimal without using a model. Use a traditional multiplication method such as distributive property or regrouping, and place the decimal point in the proper place. The process should be:

1. Estimate your solution.
2. Multiply as though you have whole numbers.
3. Place the decimal in the product.

To place the decimal you simply count the number of decimal places in each factor. If the two factors have a total of 2 decimal places then your product also will.

Example 2

Multiply 4 x $6.18.

1. Estimate by rounding the decimal to the nearest whole number.	2. Multiply as if you have two whole numbers. You will ignore the decimal for now.	3. Count the number of places after the decimal in each factor. Next, find the total number of decimal places in the product.
4 x $6 = $24 Your answer should be about $24.	3 $6.18 x 4 24 72	3 $6.18 ← 2 places after the decimal x 4 ← 0 places after the decimal + ____ 24.72 ← 2 places after the decimal

Now compare your estimate to your solution. $24.72 is close to $24 so your solution should be **$24.72**.

Example 3

Multiply 5 x 0.819 cm.

1. Estimate by rounding the decimal to the nearest whole number.	2. Multiply as if you have two whole numbers. You will ignore the decimal for now.	3. Count the number of places after the decimal in each factor. Next, find the total number of decimal places in the product.
5 x 1 cm = 5 cm Your answer should be about 5 cm.	4 4 0.819 x 5 4 095	4 4 0.819 ← 3 places after the decimal x 5 ← 0 places after the decimal + ____ 4.095 ← 3 places after the decimal

Since the estimate 5 cm is close to the product 4.095 cm you are right! Your solution is **4.095 cm**.

Let's Practice

- In your Workbook turn to Unit 1, Lesson 9 and complete 1 to 16.

Problem Solving

Sometimes the problem you need to work out is in a story or word problem. That means you will need to figure out the number sentence first. Here is a word problem that shows one method for solving the problem.

Example 4

Last week Daksha worked 3.5 hours on Wednesday and 4.5 hours on Friday. Daksha makes $7.65 per hour. How much money did Daksha earn last week?

Make a plan:

Daksha's earnings = # hours worked x $ per hour

You know: Daksha worked 3.5 hours and 4.5 hours in one week.

Daksha earns $7.65 per hour.

There are two parts:

1. Find the number of hours Daksha worked.
2. Multiply by $7.65.

The number of hours Daksha worked:

Wednesday hours + Friday hours = Number of hours worked

3.5 hrs + 4.5 hrs = Number of hours worked

$$\begin{array}{rr} & 3.5 \\ \text{Add:} & +\ 4.5 \\ \hline & 8.0 \end{array}$$

Estimate the product:

$8 per hour x 8 hours = $ 64

Multiply and place the decimal:

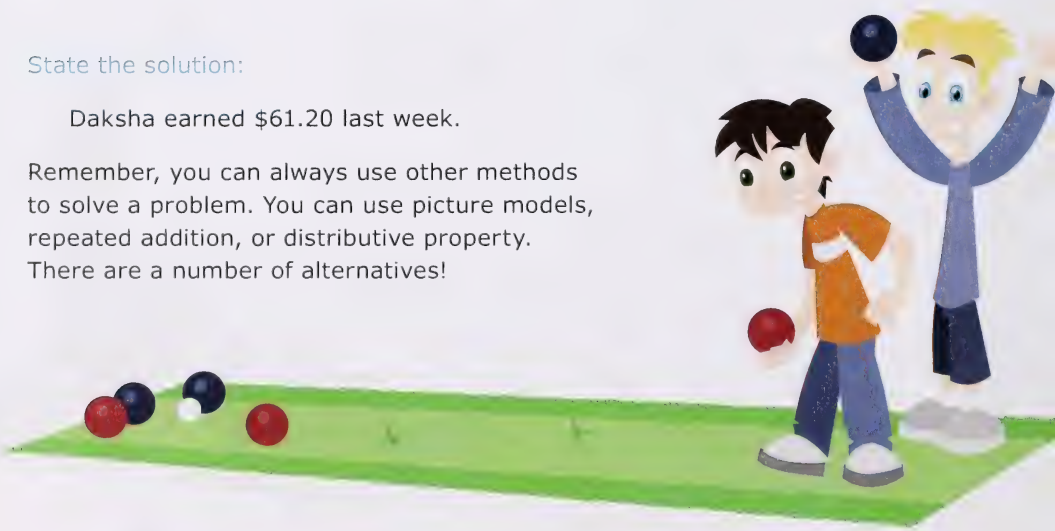
$$\begin{array}{rl} {\scriptstyle 5\ 4} & \\ 7.65 & \longleftarrow \text{2 places after the decimal} \\ \times\ \ 8 & \longleftarrow \text{0 places after the decimal} \\ \hline 61.20 & \longleftarrow \text{2 places after the decimal} \end{array}$$

The estimate of $64 is close to the product $61.20.

State the solution:

Daksha earned $61.20 last week.

Remember, you can always use other methods to solve a problem. You can use picture models, repeated addition, or distributive property. There are a number of alternatives!



Let's Explore

Exploration 2: Choosing Jobs

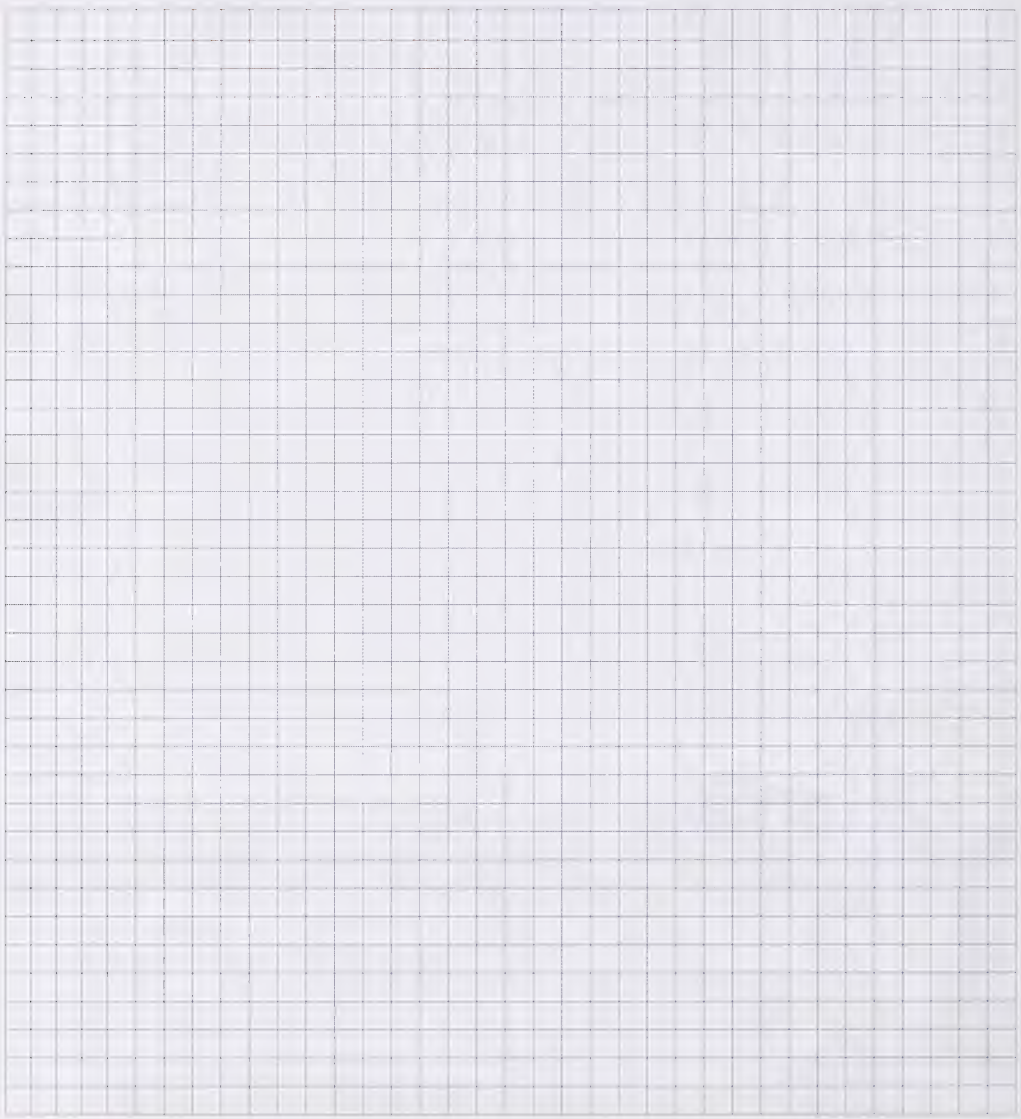
Materials: Unit 1, Lesson 9, Exploration 2 page in your Workbook, Pencil

You need to choose a job. Job A pays $8.58 per hour and Job B pays $11.32 per hour.

1. Reflect: Is it always better to take a higher paying job? Why or why not?
2. Would you choose Job A or Job B?
3. Why did you choose Job ___?
4. Would you choose Job A for 8 hours or Job B for 8 hours? Explain your reasoning.
5. Would you choose Job A for 4 hours or Job B for 7 hours? Explain your reasoning.
6. Would you choose Job A for 6 hours or Job B for 3 hours? Explain your reasoning.

Let's Practice

- In your Workbook turn to Unit 1, Lesson 9 and complete 17 to 19.



Lesson 10

Dividing Decimals

Lacrosse

Lacrosse is a game played using a solid rubber ball and a racquet with a long handle that is called a lacrosse stick. Alyssa and Zach are playing lacrosse in a league after school. The team meets twice a week and play in tournaments with other teams.

Alyssa, Zach and four of their friends bought a six-pack of fruit juice together after their lacrosse game. They each pay the same amount for the juice. If the six-pack of juice costs $4.62, how much does each person owe to the nearest cent?

This problem requires division. You have to divide the dollar amount by the number of players, which is 6.

Alyssa estimates the amount they will each have to pay:

$42 \div 6 = 7$ and $48 \div 6 = 8$

$4.20 \div 6 = 0.70$ and $4.80 \div 6 = 0.80$

She tells them they will pay between $0.70 and $0.80.

To get the final answer, Zach divides:

Cost of six-pack of juice ÷ Number of students

$4.62 \div 6$

He uses long division:

$$\begin{array}{r} 0.77 \\ 6\overline{)4.62} \\ \underline{4\ 2\ \downarrow} \\ 42 \\ \underline{-\ 42} \\ 0 \end{array}$$

Zach says that each player should pay $0.77.

Reflection

Was Alyssa's estimate close enough to Zach's answer to be true?

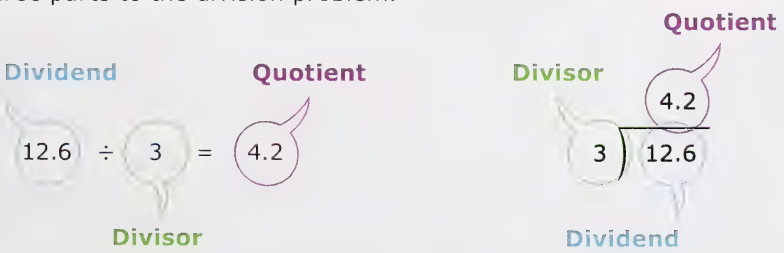
Objectives for this Lesson

In this lesson you will explore the following concepts:

- Place the decimal point in a quotient
- Predict quotients of decimals using estimation strategies
- Solve a given problem that involves division of decimals

Division Review

Long division can be used to divide a two or three digit number (**dividend**) by a one digit number (**divisor**). In long division or in symbolic form there are three parts to the division problem:



You should recall the method of long division by observing the example.

- Go online to watch the Notepad Tutor Lesson: Long Division 1-Digit Divisor by 3-Digit Dividend.

Example 1

Divide 105 ÷ 7.

1. Divide 10 by 7.
2. Multiply 1 x 7.
3. Subtract 7 from 10.
4. Bring down the 5.
5. Divide 35 by 7.
6. Multiply 5 x 7.
7. Subtract 35 – 35.

$$\begin{array}{r} 15 \\ 7\overline{)105} \\ 7\downarrow \\ 35 \\ \hline 35 \\ 0 \end{array}$$

After bringing down the last digit of the dividend and getting a remainder of 0, you have finished dividing.

You will use the same process to divide decimals. There is one extra step for dividing decimals, and that is placing the decimal place. Practice long division before you move on to decimals.

Now It's Your Turn!

Divide.

a. 324 ÷ 3

b. 310 ÷ 5

c. 126 ÷ 7

Solutions

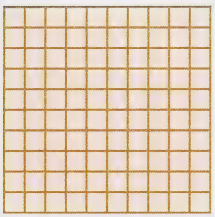
a. 108

b. 62

c. 18

Modelling Division

Base 10 Blocks are often used to represent whole numbers and decimal numbers. When using them for decimal numbers, here are the values of each type of block:

Value	1	0.1	0.01
Block			
Name	Flat	Long	Unit

To model decimal division you may use the blocks to represent the dividend.

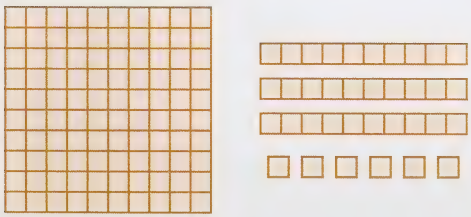
Example 2

Model 1.36 ÷ 4 to find the **quotient**.

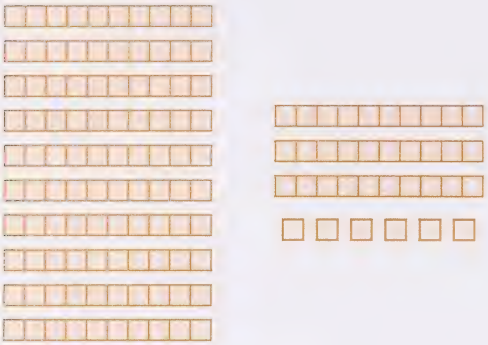
Using the Sharing Method

You need to find four equal groups.

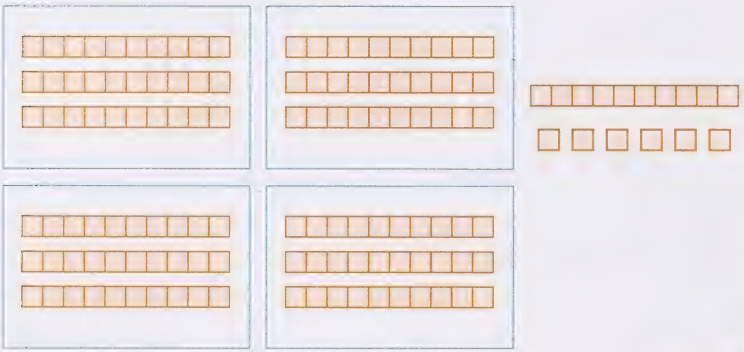
Model 1.36:



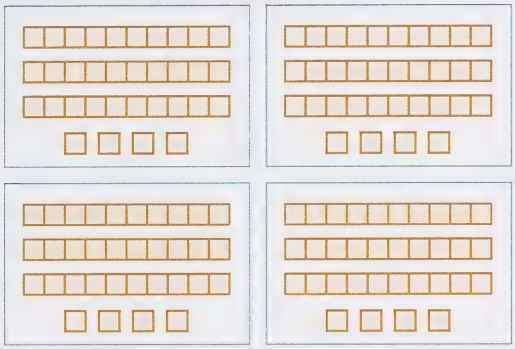
You will need to break down the 1 flat into 10 longs:



Try to make 4 equal sized groups:



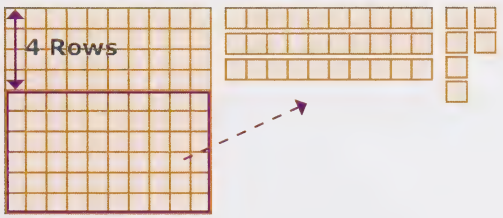
Now break down the long and share the rest of the units equally with the four groups:



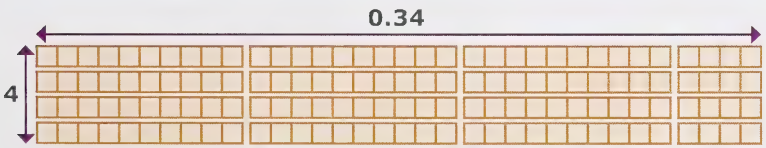
The answer is the number in each of the 4 groups: **0.34**

Using an Array

Arrange the blocks in a rectangle so that you have 4 rows:



Create the rectangle and find the measure of the side:



This side give you the quotient.

The quotient is: **0.34**

Let's Explore

Exploration 1: Modelling Decimal Division

Materials: Unit 1, Lesson 10, Exploration 1 page in your Workbook, Base 10 Block Cut-outs from the back of this Unit in your Workbook, Scissors, Pencil

Use cut-out Base 10 Blocks to model each of the following. Sketch your model and describe the method you used.

1. 8.5 ÷ 5
2. 1.62 ÷ 6
3. 0.72 ÷ 8
4. 2.72 ÷ 4
5. 1.19 ÷ 7
6. Which method worked best for each problem?
7. Reflect: What is a method you could use to solve 4.2 ÷ 3? Create a model of the problem and find the quotient.
8. Use the method you described in number 7 to solve 1.04 ÷ 4.

Decimal Division

You need to know how to place the decimal in division problems. Use what you know about decimal multiplication to help you.

Example 3

Place the decimal point in the quotient:
4.2 ÷ 7 = 6

If this was written as a multiplication problem it would be: 7 x 6 = 4.2

For the product to be 4.2, there would have to be one place after the decimal in one of the factors: 7 x 0.6 = 4.2

4.2 ÷ 7 = **0.6**

Example 4

Place the decimal point in the dividend to make the quotient correct:

18 ÷ 3 = 0.06

The multiplication sentence that gives the dividend is: 3 x 0.06 = 18

That means there should be two places behind the decimal: 3 x 0.06 = 0.18

The decimal should be placed: 0.18 ÷ 3 = 0.06

You can use estimation strategies before division. Use **compatible numbers** to estimate. Once you know the estimate, solve using division of decimals.

Example 5

Estimate the quotient of 1.47 ÷ 3 and solve using division.

Estimate:

Convert the decimal to 1.5: 1.5 ÷ 3 = 5

A compatible number sentence is: 3 x 5 = 15

Place the decimal in the estimate:

3 x 0.5 = 1.5

one place behind the decimal on each side of the equation

The estimate is 0.5

Division:

Set up for long division:	Bring the decimal up and place it above the same position it was in:	Divide 14 by 3 and multiply 4 by 3 to get 12, then subtract. Bring down the 7:	Divide 27 by 3:
$3\overline{)1.47}$	$3\overline{)1.47}$	$\begin{array}{r} .4 \\ 3\overline{)1.47} \\ 1\ 2\downarrow \\ \hline 27 \end{array}$	$\begin{array}{r} .49 \\ 3\overline{)1.47} \\ 1\ 2\downarrow \\ \hline 27 \\ \underline{27} \\ 0 \end{array}$

The quotient 0.49 is very close to the estimate 0.5.

$1.47 \div 3 = \mathbf{0.49}$

Let's Practice

- Go online to watch the Notepad Tutor Lesson: Division with Decimals (3-Digit by 1-Digit Natural Number Divisor).
- Turn in your Workbook to Unit 1, Lesson 10 and complete 1 to 18.

Problem Solving

To solve a story problem you need to identify the parts of the problem. You should make a model or picture of the problem, estimate the solution and solve the problem.

Example 6

A sailboat travels 46.5 km in 3 hours. How far does the boat travel each hour?

Draw a picture:

Write a number sentence:

$46.5 \div 3 =$

Estimate:

$48 \div 3 = 16$

Solve:

$$\begin{array}{r} 15.5 \\ 3\overline{)46.5} \\ \underline{3} \\ 16 \\ \underline{15} \\ 15 \\ \underline{15} \\ 0 \end{array}$$

The quotient 15.5 is very close to the estimate 16.

The sailboat travelled 15.5 km each hour.

Let's Practice

- Turn in your Workbook to Unit 1, Lesson 10 and complete 19 to 21.

Lesson 11

Multiplication and Division Problem Solving

Finding Averages

The running long jump in a track competition is measured from a line called the foul line. The measure ends at the first point the athlete hits the sand. Here are some running long jump distances:

Student	Distance in metres
Alyssa	3.7
Cameron	4.1
Daksha	3.9
Lian	3.5
Nina	3.8
Zach	4.2

What is the average distance for a long jump by our students?

To find an average you simply add all of the values, and divide that sum by the number of values in the list.

$$\frac{3.7 + 4.1 + 3.9 + 3.5 + 3.8 + 4.2}{6} = \frac{23.2}{6} \approx 3.9 \text{ metres}$$

The symbol ≈ means "is about". This can be used when a decimal problem calls for rounding the answer. In this case the decimal was rounded to the tenths place.

Reflection

How many canisters of oats will Alyssa and Cameron have prepared by the end of 5 hours if they pack the same number each hour? The answer to this question would be a multiple of what number?

Objectives for this Lesson

In this lesson you will explore the following concepts:

- Solve a given problem that involves multiplication and division of decimals
- Determine the reasonableness of an answer
- Determine whether the use of technology is appropriate to solve a given problem, and explain
- Use technology when appropriate to solve a given problem

Solving Problems

Sometimes the problem doesn't have a decimal, but the quotient is a decimal. See how that can happen using this problem.

Example 1

Lian had 6 metres of rope. She cut the rope into 15 equal size pieces. How long will each piece of rope be?

Draw a picture

6 m

Write a number sentence:

$6 \div 15 =$

Estimate:

$60 \div 15 = 4$ so $6.0 \div 15 =$ **0.4**

Divide:

$$\begin{array}{r} 0.4 \\ 15\overline{)6.0} \\ \underline{6\ 0} \\ 0 \end{array}$$

The answer is: Each piece of rope will be 0.4 metres.

Some problems require more than one step.

Example 2

Alyssa wants to buy two sets of stationery that cost $5.87 each. She also wants to buy a new pen for $2.35. What will be the cost of her purchase?

Draw a picture:

Estimate:

$6 + $6 + $2 = $14

Plan:

There are two ways to solve:

Repeated addition: 5.87 + 5.87 + 2.35

Multiplication and addition: 5.87 x 2 + 2.35

Solve:

$$\begin{array}{r} 5.87 \\ \times \quad 2 \\ \hline 11.74 \end{array} \qquad \begin{array}{r} 11.74 \\ + \ 2.35 \\ \hline 14.09 \end{array}$$

The cost of Alyssa's purchase will be $14.09.

- Go online to watch the Notepad Tutor Lesson: Determining if a Decimal Answer is Reasonable.

Using Technology

Sometimes problems involve very large numbers. When multiplying and dividing by more than a one digit number it is acceptable to use technology. This means that you will need a calculator.

Here are some examples of when you would use technology.

Use Paper and Pencil	Use Technology
2.4 ÷ 3	$54.87 x 28
5.67 x 8	1 254 ÷ 1.5
24.5 ÷ 5	28.97 x 2.45

Notice that you use paper and pencil when there are one digit multipliers or divisors. Calculators are used when dividing by a number with two or more digits. They are also used with products that require multiplying four, five, or more digits by two digits.

Before using technology to solve a problem, you should always estimate your solution. This simple step will ensure that you did not make any entry mistakes while using your calculator.

Example 3

$145.88 ÷ 65

Estimate:

Use compatible numbers: 140 ÷ 70 = 2

Calculate:

Use a calculator to find: 145.88 ÷ 65 ≈ 2.244 308

Round:

Since this is a money problem there should be two places after the decimal.

$145.88 ÷ 65 ≈ $2.24

Let's Explore

Exploration 1: Technology Problems

Materials: Unit 1, Lesson 11, Exploration 1 page in your Workbook, Calculator, Pencil

Write an estimate and solve the following. Do not use a calculator unless it is necessary.

1. 6 789 x 12.5
2. $475.88 x 29
3. 5 872 x 15.75
4. 82.4 x 8
5. $29.99 x 4
6. 900 x 10
7. 325 ÷ 5
8. 32 310.6 ÷ 588
9. 588 ÷ 12.25
10. 1 311.12 ÷ 54
11. 76.8 ÷ 6
12. 197.4 ÷ 3
13. List the problems you solved by using your calculator.
14. Did any of these problems have a one digit **multiplier**?
15. Did any of these problems have a one digit divisor?
16. If your answer to 14 or 15 was yes, work those problems using paper and pencil.

Example 3

Seats for the Hockey Finals will be sold at the following costs:

Section	Cost
A	$15.89
B	$29.95
C	$45.68

The fan club buys 32 seats in Section A, forty-one seats in Section B and twenty seats in Section C. How much did the fan club spend?

Picture:

32 x A 41 x 20 x C

Plan:

Multiply then add.

Calculate:

$15.89 x 32 = $508.48

$29.95 x 41 = $1 227.95

$45.68 x 20 = $913.60

$508.48 + $1 227.95 + $913.60 = $2 650.03

The fan club spent $2 650.03.

Let's Practice

- Turn in your Workbook to Unit 1, Lesson 11 and complete 1 to 12.
- Go online to watch the Notepad Tutor Lesson: Order of Operations in Problem Solving with Decimals.
- Go online to SuccessChecker and complete the Unit test to check your understanding.
- Go online to play games that will help you review your glossary terms.

Sources - Student Learning Guide

Lesson 1

Page 1-1: Population Explosion, https://www.cia.gov/library/publications/the-world-factbook/print/xx.htm; *Page 1-10:* Canada's 2007 retail trade, http://www.statcan.gc.ca/pub/11-621-m/11-621-m2008071-eng.htm; Distance to the moon, http://www.universetoday.com/guide-to-space/the-moon/distance-to-the-moon/; Video game related sales for November 2006, http://www.msnbc.msn.com/id/16115967/; Average weight of an Asian elephant, http://www.britannica.com/EBchecked/topic/184366/elephant; *Page 1-11:* Average length of a carpenter ant, http://www.insectsofalberta.com/carpenterant_3.htm; Thickness of a penny, http://www.mint.ca/store/mint/learn/circulation-currency-1100028; Wavelength of green light, http://eosweb.larc.nasa.gov/EDDOCS/Wavelengths_for_Colors.html#green

Lesson 2

Page 1-13: Theme Park Attendance, http://www.themeit.com/attendance_report2007.pdf

Lesson 5

Page 1-45: Numbers in Nature, http://www.world-mysteries.com/sci_17.htm

Lesson 6

Page 1-53: Describing Oranges, http://www.cpma.ca/en_stats_top_ten_fruits.asp

Lesson 7

Page 1-61: Water Conservation, http://www.environmentalindicators.com/htdocs/indicators/6wate.htm

Lesson 8

Page 1-71: Nutrition and Health, http://kidshealth.org/teen/food_fitness/nutrition/fat_calories.html

ISBN 978-0-7741-3097-4